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Brief paper

Distributed formation control with open-loop Nash strategy[☆]

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ABSTRACT

In this paper, we investigate game theoretical multi-agent formation control problem. The main challenge of this problem is how to enable local execution of game strategies, e.g., Nash equilibrium, which generally requires global information in the presence of communication topology among agents. Toward this, we first derive open-loop Nash equilibrium for agents minimizing formation error based performance indices and proceed to introduce a distributed estimation scheme to facilitate local implementation of the derived Nash strategies. In addition, a shrinking horizon control technique is integrated into the proposed scheme to handle unexpected state changes. The proposed scheme is fully distributed and ensures an $\epsilon-$ Nash equilibrium. An illustrative example is presented to verify the effectiveness.

1. Introduction

In the past decades, cooperative control of multi-agent systems (i.e., MAS) received increasing attention in the community, witnessed by breakthroughs in diverse applications such as formation control (Keviczky, Borrelli, Fregene, Godbole, & Balas, 2008; Olfati-Saber, 2006; Peng, Li, & Ye, 2018; Qu, 2009; Ren, Beard, & Atkins, 2007; Stipanovic, Inalhan, Teo, & Tomlin, 2004) and smart grid (Xin, Liu, Qu, & Gan, 2014; Zhang, Li, Qi, & Xin, 2017), just to name a few. Conventionally, agents in MAS are assumed to reach consensus in a collaborative way, meaning that there exists a single global performance index or potential function which these agents will optimize against (Marden, Arslan, & Shamma, 2009). However, if all agents are noncooperative in nature, then the problem becomes a non-cooperative game (Basar & Olsder, 1998) where each agent attempts to minimize/maximize against its own performance index by taking into account other players' actions (Nedic & Ozdaglar, 2009), and eventually reaches Nash equilibrium (i.e., NE) as a whole. Notable applications on game strategies among multiple entities include battlefield management (Cruz et al., 2001), air combat (Virtanen, Karelahti, & Raivio, 2006), and security among cyber physical systems (Li, Shi, Cheng, Chen, & Quevedo, 2015), just to name a few. It should be noted that, albeit their effectiveness, game strategies seeking NE will generally require each agent to possess global information of the overall group, and this requirement is stringent and often unattainable, due to the fact that each agent only has access to local information governed by the information topology. Hence, punitive action should be taken at each agent to overcome the inherent limited information and achieve NE in a distributive manner. In this paper, we attempt to solve distributed open-loop ε-NE seeking problem among networked MAS, and if successful, will greatly enhance the autonomy and resilience of each agent, thus improving the scalability of the overall network.

Our work is related to literatures on Nash games (Basar & Olsder, 1998; Isaacs, 1965), especially distributed seeking of NE. A distributed learning algorithm is proposed in Chen and Huang (2012) for finding NE in a spatial spectrum access game, albeit for games with finite action spaces; Zhu and Frazzoli (2012) studied distributed computation of generalized NE under delayed information; Frihauf, Krstic, and Basar (2012) incorporated extremum seeking principle into NE seeking, with which the gradients of cost function are estimated, similar work can be found in Stankovic, Johansson, and Stipanovic (2012), where stochastic extremum seeking scheme is introduced to estimate its own cost function as well as actions of other agents. Gharesifard and Cortés (2013) considered a distributed algorithm for NE seeking in a two-network zero-sum game. Salehisadaghiani and Pavel (2016) proposed a distributed discrete algorithm for NE seeking by estimating the action of the connected neighbors; In

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authors' previous work, pursuit-evasion game strategies are investigated in Lin, Qu, and Simaan (2014, 2015) and proved that NE can be obtained under limited observations; Lou, Hong, Xie, Shi, and Johansson (2016) studied NE seeking problem of zero-sum games with switching topologies and investigated the connection between convergence to NE and topologies as well as step sizes. Another worth mentioned games strategies are aggregative games, where the action of each system is influenced by the population of the network. In Liang, Yi, and Hong (2017), distributed NE seeking for aggregative games is verified under coupled constraints; Parise, Gentile, Grammatico, and Lygeros (2015) studied quasi-aggregative game for large population of heterogeneous systems, and proved that NE can be attained locally should graph be undirected.

Another closely related problem is game theoretical formation control problem of MAS. As is well known, multi-agent formation control has been extensively studied in the community, a comprehensive review can be found in Cao, Yu, Ren, and Chen (2013). Some of the works on incorporating game theory into the formation control problem are introduced as follows. Formation flight problem is treated as a cooperative game in Anderson and Robbins (1998). Formation control of mobile robot is investigated in Gu (2008) using open-loop NE and receding horizon strategy. Semsar-Kazerooni and Khorasani (2009) studied cooperative game strategy, specifically, Pareto strategy and its applications in consensus seeking, along with a comparison of Nash strategies.

However, it should be pointed out that, to the best of our knowledge, all of the aforementioned results are derived with rather stringent topological requirements (i.e., undirected graph), and rare work has been done to address the fundamental question about how to seek open-loop NE locally. In this paper, we attempt to tackle formation control problem using open-loop Nash strategy, and propose a systematic strategy about local implementation of open-loop NE under mild assumptions. The main contribution of this paper is two-fold: (i) we prove that open-loop NE is locally attainable with distributed estimation of terminal state vector; (ii) we demonstrate rigorously that the formation control of MAS using Nash strategies will lead to $\epsilon-$NE. The remainder of this paper is organized as follows. A brief setup of the game problem is presented in Section 2. The main results of open-loop Nash strategy and its local implementation is presented in Sections 3 to 5. An illustrative example is presented and analyzed in Section 6, and the conclusion is rendered in Section 7.

2. Problem formulation

In this paper, we consider formation control problem of a group of N homogeneous agents, and dynamics of the ith agent is described as follows:

$$\dot{x}_i(t) = A(t)x_i(t) + B(t)u_i(t) \tag{1a}$$

$$y_i(t) = C(t)x_i(t) \tag{1b}$$

where $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^m$, and $u_i \in \mathbb{R}^p$ are the state, output and input of agent i, respectively, $A(t) \in \mathbb{R}^{n\times n}$, $B(t) \in \mathbb{R}^{n\times p}$, and $C(t) \in \mathbb{R}^{m\times n}$ are time-varying system matrices. In what follows, notation of matrices $A(t)$, $B(t)$, and $C(t)$ will be simplified to A, B, and C for the sake of notation brevity. Note that initial condition $x_i(0)$ are only known to agent i.

Without loss of any generality, we assume that each agent is able to exchange information with its connected neighbors through a communication network. The network topology can be governed by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1,\ 2,\ \ldots,\ N\}$ denotes node set and $\mathcal{E}$ is the set of directed edges/paths. In particular, neighborhood set for agent i is defined as $\mathcal{N}_i \subseteq \mathcal{V}$, i.e., $(i,j) \in \mathcal{E}$ for any $j \in \mathcal{N}_i$. Since each agent only has access to local measurements and its own performance index, the underlying formation control problem can be treated as a differential game where each player has its own objectives to pursue over a finite time interval. In particular, performance index J_i for agent i is chosen as follows:

$$J_i = \sum_{j\in\mathcal{N}_i} \frac{f_{ij}}{2}\|y_i(t_f) - y_j(t_f) - \mu_{ij}\|^2_{F_f} + \frac{f_i}{2}\|y_i(t_f)\|^2_{F_o} + \int_0^{t_f} \frac{1}{2r_i}\|u_i\|^2_{R(t)}\mathrm{d}t \tag{2}$$

where $\|x\|_F = x^T F x$, μ_{ij} is a prescribed relative displacement vector, formation error weight matrix $F_f \in \mathbb{R}^{m\times m}$ is positive semi-definite, output regulation weight matrix $F_o \in \mathbb{R}^{m\times m}$ is positive semi-definite, $R(t) \in \mathbb{R}^{p\times p}$ is time-varying, positive definite and shorted for R hereafter for the notation brevity, and f_{ij}, f_i, and r_i are positive scalars. Obviously, in order to minimize J_i, the desired input u_i should achieve three objectives: (i) minimizing the formation error, i.e., first term in (2), (ii) minimizing its total control energy consumed in the process, i.e., third term in (2), and (iii) regulating its own output, i.e., second term in (2) where regulation matrix F_o should have relatively smaller eigenvalues than F_f in order to achieve formation control. In what follows, the following assumption is made regarding the connectivity of graph $\mathcal{G}$.

Assumption 1. Graph $\mathcal{G}$ is assumed to be directed and at least connected.[1]

Remark 1. Note that even in the extreme cases where graph $\mathcal{G}$ is completely isolated, the underlying game is still mathematically valid and simply degenerated to the standard optimization problem of J_i. Therefore, Assumption 1 on graph connectivity is poised to instigate the Nash game among MAS so that the agents can interact with each other.

Before proceeding further, the following lemma is introduced. Its conclusion will serve as a basis in the rest of the development.

Lemma 1. *Given $F = \mathrm{diag}\{f_1, \ldots, f_N\} \otimes F_o$, weighted Laplacian matrix $L = [L_{ij}] \in \mathbb{R}^{N\times N}$ with*

$$L_{ij} = \begin{cases} -f_{ij} & \text{if } j \in \mathcal{N}_i \\ 0 & \text{if } j \notin \mathcal{N}_i \\ \sum_{j\in\mathcal{N}_i} f_{ij} & \text{if } j = i \end{cases}, \tag{3}$$

and $D_{t_2t_1} = \mathrm{diag}\{r_1, \ldots, r_N\} \otimes \bar{D}_{t_2t_1}$ with

$$\bar{D}_{t_2t_1} = C_f W_{t_2t_1} C_f^T \tag{4}$$

where $C_f = C(t_f)$, $0 \le t_1 \le t_2 \le t_f$,

$$W_{t_2t_1} = \int_{t_1}^{t_f} \Phi(t,\tau)BR^{-1}B^T\Phi^T(t_f,\tau)d\tau, \tag{5}$$

and Φ is the state-transition matrix of A. Then, eigenvalues of

$$H_{t_2t_1} \triangleq I + D_{t_2t_1}(F + L \otimes F_f) \tag{6}$$

have positive real parts, where I denotes identity matrix of proper dimension and $\otimes$ is the Kronecker product.

[1] A graph is said to be at least connected if it contains at least one globally reachable node (Li & Qu, 2014).

Proof. Based on associative and bilinear properties of Kronecker product, (6) can be expressed as

$$\begin{aligned}H_{t_2t_1} =& I + \left(\mathrm{diag}\{r_1,\dots,r_N\}\otimes \bar{D}_{t_2t_1}\right)(\\ &\quad \mathrm{diag}\{f_1,\dots,f_N\}\otimes F_o + L\otimes F_f)\\ =& I + (\mathrm{diag}\{r_1f_1,\dots,r_Nf_N\}L)\otimes\left[\bar{D}_{t_2t_1}(F_o+F_f)\right]\end{aligned}$$

Firstly, given positive scalars r_i and f_i and Laplacian matrix L, product $(\mathrm{diag}\{r_1f_1,\dots,r_Nf_N\}L)$ is also a weighted Laplacian matrix and its eigenvalues have non-negative real parts simply according to Gershgorin circle theorem. Secondly, since F_o and F_f are both positive semi-definite, their addition (F_o+F_f) is also positive semi-definite. Moreover, since it is obvious that $\bar{D}_{t_2t_1}$ in (4) is positive semi-definite, eigenvalues of product $\bar{D}_{t_2t_1}(F_o+F_f)$ are real and non-negative.[2] Finally, according to its spectrum property, eigenvalues of Kronecker product $(\mathrm{diag}\{r_1f_1,\dots,r_Nf_N\}L)\otimes[\bar{D}_{t_2t_1}(F_o+F_f)]$ are the eigenvalue products between $(\mathrm{diag}\{r_1f_1,\dots,r_Nf_N\}L)$ and $[\bar{D}_{t_2t_1}(F_o+F_f)]$ and hence have non-negative real parts. Therefore, eigenvalues of matrix $H_{t_2t_1}$ in (6) have positive real parts. ■

In this paper, we attempt to solve open-loop (where control inputs are functions of initial states and time only) formation problem using differential game strategy. Special attention will be paid on how to execute open-loop NE strategy in a distributive manner. The concept of open-loop NE is defined as follows:

Definition 1. For the system and performance index defined in (1) and (2), strategies $u_1^*,\dots,u_N^*$ form an open-loop NE if inequality

$$J_i(u_1^*,\dots,u_i^*,\dots,u_N^*)\le J_i(u_1^*,\dots,u_i,\dots,u_N^*) \tag{7}$$

holds for $u_i\in U_i$ for all $i=1,\dots,N$, where $U_i=\{u_i(t,x(0))\in\mathbb{R}^p \mid t\in[0,t_f]\}$ is the open-loop control set for agent i.

The NE solution is widely used in non-cooperative game scenarios and is defined as an equilibrium where it is impossible for any agent to lower its performance index value by unilaterally deviating from its Nash strategy. However, it should be pointed out that, albeit NE exists in theory, its value is difficult to attain in practice, and in some cases the maximum benefit for each agent to deviate from its NE is $\epsilon\ge 0$, and its corresponding equilibrium is defined as ϵ-NE. Formally,

Definition 2. For the system and performance indices defined in (1) and (2), strategies $u_1^*,\dots,u_N^*$ for all $i=1,\dots,N$ form an open-loop ϵ-NE if inequality

$$J_i(u_1^*,\dots,u_i^*,\dots,u_N^*)\le J_i(u_1^*,\dots,u_i,\dots,u_N^*)+\epsilon \tag{8}$$

holds for $u_i\in U_i$ for all $i=1,\dots,N$ and scalar $\epsilon\ge 0$ may be a function of initial conditions.

3. Open-loop Nash equilibrium

The following theorem summarizes the existence and uniqueness of NE associated with the underlying multi-agent formation control problem.

Theorem 1. *For a group of N agents with dynamics defined in (1) and performance indices in (2), there exists an open-loop NE given by*

$$\begin{aligned}u_i^* =& -r_iR^{-1}B^T\Phi^T(t_f,t)C_f^T(e_i^T\otimes I)\Big\{-\mu+\\ &\left(F+L\otimes F_f\right)H_{t_f0}^{-1}\left[\Phi_C(t_f)x(0)+D_{t_f0}\mu\right]\Big\}\end{aligned} \tag{9}$$

for all $i=1,\dots,N$, where e_i is a $N\times 1$ vector of zeros except its ith entry being 1, H_{t_f0} is defined in (6), $\Phi_C(t_f)=I_N\otimes\left[C_f\Phi(t_f,0)\right]$, $x(0)=[x_1^T(0),\dots,x_N^T(0)]^T$, and

$$\mu=\left[\sum_{j\in\mathcal{N}_i} f_{1j}F_f\mu_{1j}^T,\cdots,\sum_{j\in\mathcal{N}_N} f_{Nj}F_f\mu_{Nj}^T\right] \tag{10}$$

Proof. Since performance index (2) is quadratic and hence strictly convex for all admissible open-loop control u_i and for all $x_i(0)$, every solution satisfying the following first order necessary conditions is sufficiently an open-loop NE (Basar & Olsder, 1998):

$$\dot{x}_i=\frac{\partial\bar{H}_i}{\partial\lambda_i}=Ax_i+Bu_i, \tag{11a}$$

$$\dot{\lambda}_i=-\frac{\partial\bar{H}_i}{\partial x_i}=-A^T\lambda_i, \tag{11b}$$

$$\lambda_i(t_f)=C_f^T\left\{f_iF_oy_i(t_f)+\sum_{j\in\mathcal{N}_i}f_{ij}F_f\left[y_i(t_f)-y_j(t_f)-\mu_{ij}\right]\right\} \tag{11c}$$

$$\frac{\partial\bar{H}_i}{\partial u_i}=\frac{1}{r_i}Ru_i+B^T\lambda_i=0, \tag{11d}$$

for all $i=1,\dots,N$, where $\bar{H}_i$ is the Hamiltonian for agent i and $\bar{H}_i=\frac{1}{2r_i}\|u_i\|_R^2+\lambda_i^T(Ax_i+Bu_i)$ and λ_i is the Lagrangian multiplier. Solving ordinary differential equation (11b) and substituting the solution into (11d) yields

$$u_i(t)=-r_iR^{-1}B^T\Phi^T(t_f,t)\lambda_i(t_f). \tag{12}$$

Substituting the above input into (11a) and solving the resulting ordinary differential equation yields

$$x_i(t_f)=\Phi(t_f,0)x_i(0)-r_iW_{t_f0}\lambda_i(t_f),$$

where W_{t_f0} is defined in (5). Multiplying C_f on both sides of the above relation and substituting (11c) into it, we obtain

$$\begin{aligned}y_i(t_f)=& C_f\Phi(t_f,0)x_i(0)-r_i\bar{D}_{t_f0}\Big\{f_iF_oy_i(t_f)+\\ &\sum_{j\in\mathcal{N}_i}f_{ij}F_f[y_i(t_f)-y_j(t_f)-\mu_{ij}]\Big\},\end{aligned}$$

where $\bar{D}_{t_f0}$ is defined in (4). Stacking the above equation for $i=1,\dots,N$ yields

$$y(t_f)=\Phi_C(t_f)x(0)-D_{t_f0}\left[(F+L\otimes F_f)y(t_f)-\mu\right] \tag{13}$$

where $y=[y_1^T,\dots,y_N^T]^T$, μ is defined in (10), and matrices D_{t_f0}, F, and L are all defined in Lemma 1. It is straightforward to solve $y(t_f)$ from the above equation as

$$y(t_f)=H_{t_f0}^{-1}\left\{\Phi_C(t_f)x(0)+D_{t_f0}\mu\right\} \tag{14}$$

It should be pointed out that $H_{t_f0}^{-1}$ exists since its eigenvalues all have positive real parts, as proved in Lemma 1. Then, substituting $y(t_f)$ into (11c) and consequently submitting the resulted $\lambda_i(t_f)$ to (12) yields open-loop strategy (9), which concludes the proof. ■

Note that, albeit the proposed strategy is deemed to be open-loop, control protocol (9) is dynamical since state-transition matrix $\Phi(t_f,t)$ is time-varying. In addition, $\lambda_i(t_f)$ is resilient to information topology as well as terminal state (i.e., $y_i(t_f)$), and as will be demonstrated later, local implementation of open-loop Nash strategy (9) becomes possible if $y_i(t_f)$ can be retrieved locally.

[2] If matrix A is positive semi-definite, it can be expressed as $A=A_s^TA_s$ for some positive semi-definite matrix A_s. Due to the fact that eigenvalues of AB are the same as BA, eigenvalues of $AB=A_s^TA_sB$ are the same as $A_sBA_s^T$ and hence non-negative if B is positive semi-definite.

4. Distributed estimation of Nash equilibrium

It is obvious that implementation of open-loop Nash strategy (9) is complicated and requires global knowledge on initial conditions and overall topological structures. Namely, agent i is dictated to possessing at least the knowledge of initial state vector $x(0)$, which is excessive and conflicts with communication infrastructure of MAS. In this section, we demonstrate that, albeit (9) seems complex in theory, the proposed Nash strategy can be greatly simplified in practice and executed locally as a result. More specifically, we will explore the possibility of estimating terminal state (i.e., $y_i(t_f)$) at each agent through a distributed observer, and then proceed to prove that the proposed Nash strategy is locally attainable.

Specifically, we propose the following observer at agent i

$$\dot{\hat{y}}_{if} = g_i\Big\{C^T\Phi(t_f,0)x_i(0) - \hat{y}_{if} - r_i\bar{D}_{t_f0}\Big[f_iF_o\hat{y}_{if} + \sum_{j\in\mathcal{N}_i} f_{ij}F_f(\hat{y}_{if} - \hat{y}_{jf} - \mu_{ij})\Big]\Big\} \tag{15}$$

for all $i = 1,\ldots,N$, where $\hat{y}_{if}(t)$ is the estimated $y_i(t_f)$ at time $t \in [0, t_f]$ and $g_i > 0$ is the estimator gain. Performance of proposed observer (15) is summarized as follows:

Theorem 2. *For a group of N agents with dynamics defined in (1) and performance indices in (2), suppose*

$$\hat{u}_i^* = -r_iR^{-1}B^T\Phi^T(t_f,t)C_f^T\Big\{f_iF_o\hat{y}_{if} + \sum_{j\in\mathcal{N}_i} f_{ij}F_f\left[\hat{y}_{if} - \hat{y}_{jf} - \mu_{ij}\right]\Big\}, \tag{16}$$

where $\hat{y}_{if}$ is the observer state specified in (15). Let us define the discrepancy between (9) and (16) as

$$\Delta u_i(t) \triangleq \hat{u}_i^*(t) - u_i^*(t). \tag{17}$$

Then, under Assumption 1, system (17) is globally exponential stable for all $i = 1,\ldots,N$, and its convergence is upper bounded by

$$\|\Delta u_i(t)\|_2 \le K_i(t)e^{-\alpha t}\|\Delta y_f(0)\|_2 \tag{18}$$

for $t \in [0, t_f]$, where $\|\cdot\|_2$ denotes the Euclidean norm, $K_i(t)$ is a positive scalar function of t to be specified,

$$\alpha = \min_j Re[\lambda_j(GH_{t_f0})], \tag{19}$$

and $Re[\lambda_j(GH_{t_f0})]$ is the real parts of the jth eigenvalue of GH_{t_f0} given $G = \mathrm{diag}\{g_1,\ldots,g_N\}\otimes I$.

Proof. Stacking (15) for all $i = 1,\ldots,N$ yields

$$\dot{\hat{y}}_f = G\big\{\Phi_C(t_f)x(0) - \hat{y}_f - D_{t_f0}\left[F\hat{y}_f + (L\otimes F_f)\hat{y}_f - \mu\right]\big\} \tag{20}$$

where $\hat{y}_f = [\hat{y}_{1f}^T,\ldots,\hat{y}_{Nf}^T]^T$. According to (13), we obtain

$$\Phi_C(t_f)x(0) = y(t_f) + D_{t_f0}\left[Fy(t_f) + (L\otimes F_f)y(t_f) - \mu\right]$$

Substituting the above equation into (20) yields

$$\dot{\hat{y}}_f = -GH_{t_f0}\left[\hat{y}_f(t) - y(t_f)\right]$$

Let $\Delta y_f(t) \triangleq \hat{y}_f(t) - y(t_f)$, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta y_f(t) = -GH_{t_f0}\Delta y_f(t) \tag{21}$$

Since G is diagonal with positive diagonal entries and eigenvalues of H_{t_f0} have positive real parts, system (21) is asymptotically stable and

$$\Delta y_f(t) = e^{-GH_{t_f0}t}\Delta y_f(0). \tag{22}$$

Taking Euclidean norm on both sides of the above relation, we obtain

$$\|\Delta y_f(t)\|_2 \le e^{-\alpha t}\|\Delta y_f(0)\|_2, \tag{23}$$

Consequently, substituting (12) and (16) together with (11c) into (17), we have

$$\Delta u_i(t) = -r_iR^{-1}B^T\Phi^T(t_f,t)C_f^T(e_i^T\otimes I)\,(F + L\otimes F_f)\Delta y_f(t)$$

Again, taking Euclidean norm on both sides of the above equation and substituting (23) into it, yields

$$\|\Delta u_i(t)\|_2 \le \Big\| - r_iR^{-1}B^T\Phi^T(t_f,t)C_f^T(e_i^T\otimes I)(F + L\otimes F_f)\Big\|_2\|e^{-(GH_{t_f0})t}\|_2\|\Delta y_f(0)\|_2$$

Let $K_i(t) \triangleq \| - r_iR^{-1}B^T\Phi^T(t_f,t)C_f^T(e_i^T\otimes I)(F+L\otimes F_f)\|_2$. Hence, it can be easily verified that $K_i(t)$ is bounded. Then, after several algebraic manipulations, (18) can be ensured, which completes the proof of Theorem 2. ■

Remark 2. According to the theorem, $\hat{u}_i^*$ can be in fact regarded as an estimate of u_i^* made by agent i. Through exchanging this estimate information among themselves according to Eq. (15), all agents are able to collaboratively make their estimates asymptotically converge to their open-loop Nash strategies. Implementing Eq. (15) is distributed in the sense that it only requires information received from local agents. Specifically, at time t, agent i only needs to send out its own information $\hat{y}_{if}(t)$ to agent j for all $j\in\mathcal{N}_i$ and receive $\hat{y}_{jf}(t)$ from agent j for all $j\in\mathcal{N}_i$.

Remark 3. It should be pointed out that the proposed terminal state observer (15) and Nash strategy estimator (16) constitute a fully decentralized Proportional–Integral dynamical system. In particular, the convergence rate of its inner loop is mainly determined by the gain g_i, while the outer loop depends on the eigen-structure of matrix (GH_{t_f0}). That is, each agent collaboratively estimates u_i^* in an asymptotical manner, such that the execution of overall open-loop NE becomes local. However, it should be pointed out that, though convergence of observer (15) is asymptotical, convergence of Δu_i is upper bounded by an exponentially decay function. In other words, the estimated NE (16) is also exponentially stable with respect to the true NE.

Remark 4. Instead of implementing (15) and (16) simultaneously, an alternative way is to let all agents communicate for a while until a satisfactory convergent of (15) is reached before the game starts. Such approach is often regarded as an offline implementation, and it could be instrumental if a sufficiently accurate NE is deemed necessary.

Theorem 3. *For a group of N agents whose dynamics governed by (1) and performance indices defined in (2). Then the open-loop NE strategies defined in (16) with observer (15) form an ϵ-NE with*

$$\epsilon = \max_i \beta_i\|\Delta y_f(0)\|_2^2 \tag{24}$$

$$\beta_i = \frac{1}{2}\left[\Big(\lambda_{\max}^{reg}f_i + \sum_{j\in\mathcal{N}_i} f_{ij}\lambda_{\max}^{form}\Big)e^{-2\alpha t_f} + r_i\lambda_{\max}^R\int_0^{t_f}K_i^2(t)e^{-2\alpha t}dt\right] \tag{25}$$

where $\lambda_{\max}^{reg}$, $\lambda_{\max}^{form}$, and $\lambda_{\max}^R$ are the maximum eigenvalues of F_o, F_f, and R, respectively.

Proof. Supposing that all agents are implementing strategy (16) with observer (15) except agent i, the optimal control for agent i can then be derived from conditions (11) as

$$u_i^*(t) = -r_i R_i^{-1} B^T \Phi^T(t_f, t) C_f^T \Bigg\{ f_i F_o y_i^*(t_f) + \sum_{j\in\mathcal{N}_i} f_{ij} F_f \Big[y_i^*(t_f) - y_j(t_f) - \mu_{ij} \Big] \Bigg\} \tag{26}$$

where $y_i(t_f)$ and $y_j^*(t_f)$ are terminal states corresponding to control $u_i(t)$ and $\hat{u}_j^*(t)$, respectively. Let $\Delta x_i(t) = x_i(t) - x_i^*(t)$, we have

$$\Delta \dot{x}_i = A_i \Delta x_i + B_i \Delta u_i, \quad \Delta x_i(0) = x_i(0) - x_i^*(0) = 0.$$

Solving the above ordinary differential equation yields

$$\Delta x_i(t_f) = \int_0^{t_f} \Phi(t_f, t) B(t) \Delta u_i(t) \mathrm{d}t \tag{27}$$

Multiplying both sides of (27) by C_f yields

$$\Delta y_i(t_f) = \int_0^{t_f} C_f \Phi(t_f, t) B(t) \Delta u_i(t) \mathrm{d}t \tag{28}$$

Suppose the value of (2) under input $\hat{u}_i^*(t)$ in (16) with observer (15) be J_i, and its counterpart under input $u_i^*(t)$ in (26) be J_i^*, we have

$$\begin{aligned} J_i - J_i^* = &\frac{1}{2} \sum_{j\in\mathcal{N}_i} f_{ij} \|\Delta y_i(t_f)\|_{F_f}^2 + \frac{1}{2} f_i \|\Delta y_i(t_f)\|_{F_o}^2 \\ &+ \sum_{j\in\mathcal{N}_i} f_{ij} \Delta y_i(t_f)^T F_f \left[y_i^*(t_f) - y_j(t_f) - \mu_{ij} \right] \\ &+ f_i \Delta y_i(t_f)^T F_o y_i^*(t_f) \\ &+ r_i \int_0^{t_f} \left(\Delta u_i^T R u_i^* + \frac{1}{2} \|\Delta u_i\|_R^2 \right) \mathrm{d}t \end{aligned}$$

Substituting (26) and (28) into the above equation yields

$$\begin{aligned} J_i - J_i^* = &\frac{1}{2} \sum_{j\in\mathcal{N}_i} f_{ij} \|\Delta y_i(t_f)\|_{F_f}^2 + \frac{1}{2} f_i \|\Delta y_i(t_f)\|_{F_o}^2 + \\ &\frac{r_i}{2} \int_0^{t_f} \|\Delta u_i\|_R^2 \mathrm{d}t \end{aligned}$$

That is,

$$\begin{aligned} J_i - J_i^* \le &\left(\frac{f_i \lambda_{\max}^{reg} + \sum_{j\in\mathcal{N}_i} f_{ij} \lambda_{\max}^{form}}{2} \right) \|\Delta y_i(t_f)\|^2 \\ &+ \frac{r_i \lambda_{\max}^R}{2} \int_0^{t_f} \|\Delta u_i\|^2 \mathrm{d}t. \end{aligned}$$

Substituting (18) and (23) into the above equation yields

$$J_i - J_i^* \le \left\{ \left(\frac{f_i \lambda_{\max}^{reg} + \sum_{j\in\mathcal{N}_i} f_{ij} \lambda_{\max}^{form}}{2} \right) e^{-2\alpha t_f} + \frac{r_i \lambda_{\max}^R}{2} \int_0^{t_f} K_i^2(t) e^{-2\alpha t} \mathrm{d}t \right\} \|\Delta y_f(0)\|_2^2$$

After several algebraic manipulations, we obtain

$$J_i - J_i^* \le \beta_i \|\Delta y_f(0)\|_2^2$$

Therefore, if ϵ is chosen according to (24), the above inequality assures that the proposed distributive open-loop Nash strategies (15) and (16) form an ϵ-NE. ■

Remark 5. Since β_i in (25) is clearly inversely proportional to α and eigenvalues of G. Therefore, it is only intuitive to choose a large $g_1, \ldots, g_N$ such that ϵ in (24) can be made sufficiently small and ϵ-NE arbitrarily close to NE in (9). However, using large rates might not be practically feasible due to peaking-like phenomenon in high-gain observers caused by large initial estimation error, i.e., $\Delta y_f(0)$. In other words, it can be concluded that choice of g_i actually preserves a tradeoff between performance of the proposed distributed strategies (i.e.,(15)–(16)) and system stability, and a mildly large gain g_i will be strongly recommended.[3]

Remark 6. Although the proposed scheme is developed within finite time horizon, it can be applied to solve infinite horizon problem as well. In particular, suppose that the performance index for an infinite horizon optimal control problem is

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt, \tag{29}$$

subject to (1). In this case, the following alternative index can be considered

$$J = x^T(t_f) F x(t_f) + \int_0^{t_f} u^T R u dt, \tag{30}$$

Although the preceding index does not regulate transient response, the resulting control is stabilizing in the sense that $x(t_f)$ is driven to be within a small neighborhood around the origin shall $F > 0$ be sufficiently large. In this regard, the underlying infinite horizon problem can be converted and subsequently solved by the proposed finite horizon approach, by simply choosing a sufficiently large t_f.

5. Shrinking horizon Nash strategy

As previous stated, distributive seeking of open-loop NE is plausible by taking advantage of information flow among networked agents. However, albeit its effectiveness, open-loop strategy is inherently inadequate when coping with dynamical scenarios due to the lack of time consistency. Hence, the proposed open-loop strategy should be augmented in order to accommodate dynamical scenarios. In this section, we propose a sample data based shrinking horizon Nash strategy that resets the open-loop strategy over a series of predefined time intervals, similar to the approximate closed-loop Nash strategy in Simaan and Cruz (1973).

Suppose the state X_i and output Y_i are vectors of x_i and y_i with sampled data at $(n-1)$ time instants between t_0 and t_f, that is

$$\begin{aligned} X_i &= \begin{bmatrix} x_i(0) & x_i(t_1) & x_i(t_2) & \cdots & x_i(t_{n-1}) & x_i(t_f) \end{bmatrix}^T, \\ Y_i &= \begin{bmatrix} y_i(0) & y_i(t_1) & y_i(t_2) & \cdots & y_i(t_{n-1}) & y_i(t_f) \end{bmatrix}^T. \end{aligned}$$

Time instances $t_0 = 0, t_1, \ldots, t_{n-1}, t_n = t_f$ are known by all agents. Before proceeding further, performance index is revised as follows to reflect the shrinking horizon at sample time instance k:

$$\begin{aligned} J_{ik} = &\sum_{j\in\mathcal{N}_i} \frac{f_{ij}}{2} \|y_i(t_f) - y_j(t_f) - \mu_{ij}\|_{F_f}^2 \\ &+ \frac{f_i}{2} \|y_i(t_f)\|_{F_o}^2 + \int_{t_k}^{t_f} \frac{1}{2r_i} \|u_{ik}\|_R^2 \mathrm{d}t \end{aligned} \tag{31}$$

for $i = 1, \ldots, N$ and $k = 0, 1, \ldots, (n-1)$. It is straightforward to find the open-loop NE at each time instance. The result is summarized into the following corollary, its proof follows directly from that of Theorem 1 and thus omitted here for the sake of brevity.

[3] We owe this observation to an anonymous reviewer.

Corollary 1. *For a group of N agents with dynamics (1) and performance index (31), under Assumption 1, the open-loop NE at time instant k for agent i is*

$$u_{ik}^{*} = -r_i R^{-1} B^T \Phi^T(t_f, t) C_f^T \Big\{ f_i F_o y_{ifk} + \sum_{j\in\mathcal{N}_i} f_{ij} F_f \left[y_{ifk} - y_{jfk} - \mu_{ij} \right] \Big\} \tag{32}$$

for $k = 1, \ldots, (n-1)$, where y_{ifk} is the state at terminal time t_f for the shrinking horizon $[t_k, t_f]$ and satisfies

$$y_{ifk} = C_f \Phi(t_f, t_{k-1}) x_i(t_{k-1}) - r_i C_f W_{t_f t_{k-1}} C_f^T \Big\{ f_i F_o y_{ifk} + \sum_{j\in\mathcal{N}_i} f_{ij} F_f [y_{ifk} - y_{jfk} - \mu_{ij}] \Big\} \tag{33}$$

$$x_i(t_k) = \Phi(t_k, t_{k-1}) x_i(t_{k-1}) - r_i W_{t_k t_{k-1}} C_f^T \Big[f_i F_o y_{ifk} + \sum_{j\in\mathcal{N}_i} f_{ij} F_f (y_{ifk} - y_{jfk} - \mu_{ij}) \Big], \tag{34}$$

where $W_{t_k t_{k-1}}$ is defined in (5). □

It is apparent that Corollary 1 ensures (32) is the open-loop NE along horizon $[t_k,\ t_f]$. Therefore, in order to implement (32) locally, we need a sequence of terminal state estimates $\hat{y}_{if1}, \ldots,\ \hat{y}_{if(n-1)}$ at time instance $0, t_1, \ldots, t_{n-1}$, as opposed to Theorem 2. The following lemma summarizes the distributed estimation of u_{ik}^{*}.

Theorem 4. *Given*

$$\hat{u}_{ik}^{*} = -r_i R^{-1} B^T \Phi^T(t_f, t) C_f^T \Big\{ f_i F_o \hat{y}_{if,k} + \sum_{j\in\mathcal{N}_i} f_{ij} F_f \left[\hat{y}_{if,k} - \hat{y}_{jf,k} - \mu_{ij} \right] \Big\}, \tag{35}$$

for $k = 1, \ldots, (n-1)$ where

$$\dot{\hat{y}}_{ifk} = g_i \Big\{ C_f \Phi(t_f, t_{k-1}) \hat{x}_i(t_{k-1}) - \hat{y}_{ifk} - r_i C_f W_{t_f t_{k-1}} C_f^T \Big[f_i F_o \hat{y}_{ifk} + \sum_{j\in\mathcal{N}_i} f_{ij} F_f (\hat{y}_{ifk} - \hat{y}_{jfk} - \mu_{ij}) \Big] \Big\} \tag{36}$$

and $\hat{x}_i(t_k)$ is defined in (34) with y_{ifk} substituted by $\hat{y}_{ifk}$, then, under the Assumption 1, $\hat{y}_{ifk}$ converges to y_{ifk} and $\hat{u}_{ik}^{*}$ converges to the Nash strategy u_{ik}^{*} in (32).

Proof. Stacking (34) from $1, \ldots, N$ and solving the resulting equation recursively yields

$$x(t_k) = [I_N \otimes \Phi(t_k, 0)] x(0) - \sum_{l=1}^{k} \left[\mathrm{diag}\{r_1, \ldots, r_N\} \otimes (W_{t_k t_{l-1}} C_f^T) \right] \cdot [(F + L \otimes F_f) y_{fl} - \mu]$$

where $x(t_k) = [x_1^T(t_k),\ x_2^T(t_k), \ldots,\ x_N^T(t_k)]^T$ amd $y_{fj} = [y_{1fj}^T,\ y_{2fj}^T, \ldots,\ y_{nfj}^T]^T$. Stacking equation (36) for $i = 1, \ldots, N$ and substituting the above equation into it with x replaced by $\hat{x}$ and y replaced by $\hat{y}$ yields

$$\dot{\hat{y}}_{fk} = G \Big\{ \Phi_C(t_f) x_0 - \hat{y}_{fk} - \sum_{l=1}^{k} D_{t_f t_{l-1}} \left[(F + L \otimes F_f) \hat{y}_{fl} - \mu \right] \Big\} \tag{37}$$

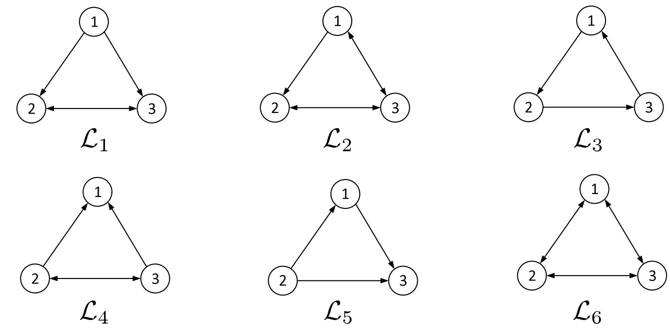

Fig. 1. Communication graph.

Rewriting the above equations into compact form yields

$$\dot{\hat{y}} = (I_N \otimes G) \big\{ \mathbf{1}_N \otimes [\Phi_C(t_f) x_0] - (I + \mathcal{M}) \hat{y} + \mathcal{M} \mu \big\} \tag{38}$$

$$\mathcal{M} = \begin{bmatrix} D_{t_f 0} & & & & \\ D_{t_f 0} & D_{t_f t_1} & & & \\ \vdots & & \ddots & & \\ D_{t_f 0} & \cdots & D_{t_f t_{n-3}} & D_{t_f t_{n-2}} & \\ D_{t_f 0} & \cdots & \cdots & D_{t_f t_{n-2}} & D_{t_f t_{n-1}} \end{bmatrix} \tag{39}$$

where $\hat{y} = [\hat{y}_{f1}^T, \ldots, \hat{y}_{fn}^T]^T$. Then, it is clear that, under the Assumption 1, all eigenvalues of lower block triangular matrix $\mathcal{M}$ have non-negative real parts. Hence, linear system (38) is asymptotically stable, and consequently $\hat{y}_{if,k}$ will converge to $y_i(t_f)$. As a result, the protocol $\hat{u}_{ik}^{*}$ in (35) converges to the open-loop Nash strategy u_{ik}^{*} in (32) asymptotically, which concludes the proof. ■

In theory, compared with the open-loop protocol, the proposed shrinking horizon strategy is strongly time consistent, and this will preserve the consistency with a set of time-varying performance indices. In practice, execution of the strategies (35) and (36) dictate agent i exchanging a vector of k-dimension $[\hat{y}_{if1}^T, \ldots, \hat{y}_{ifk}^T]^T$ with its connected neighbors at time instance k. Analogously, convergence of (36) depends on the value of g_i, and it can be easily verified that the proposed scheme also forms an ϵ-Nash strategy.

6. Illustrative example

In this section, we consider a simple formation control example in two-dimensional space and examine the proposed distributed open-loop Nash strategies. The dynamics are assumed to be double integrator, and for agent i we have,

$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i, \tag{40}$$

where p_i and v_i are states of agent i, u_i is the input to agent i. For the ease of illustration, the horizontal axis is called the x-axis and the vertical axis is called the y-axis.

In what follows, we consider a formation of three agents with communication topology shown in Fig. 1, where $\mathcal{L}_1$ illustrates a connected digraph and $\mathcal{L}_2$ to $\mathcal{L}_6$ depict strongly connected digraphs.

In addition, the Laplacian matrices for $\mathcal{L}_1$ to $\mathcal{L}_6$ can be obtained straightforward from (3), and the initial states of the agents are given by

$$p_1 = [0\ \ 0]^T,\ p_2 = [-3\ \ 0]^T,\ p_3 = [3\ \ 0]^T,$$
$$v_1 = [0\ \ 2]^T,\ v_2 = [0\ \ 0]^T,\ v_3 = [0\ \ 0]^T,$$

Table 1
Comparison of total terminal errors.

	Original Nash	D-Nash ($g = 10$)	D-Nash ($g = 20$)	SH-Nash ($g = 10$)	SH-Nash ($g = 20$)
Ω ($\mathcal{L}_1$)	0.0195	0.1235	0.0753	0.0974	0.0164
Ω ($\mathcal{L}_2$)	0.0148	0.1099	0.0615	0.0146	0.0137
Ω ($\mathcal{L}_3$)	0.019	0.1101	0.0621	0.066	0.0203
Ω ($\mathcal{L}_4$)	0.0146	0.1097	0.0613	0.0214	0.0055
Ω ($\mathcal{L}_5$)	0.0143	0.1171	0.0697	0.098	0.0151
Ω ($\mathcal{L}_6$)	0.0115	0.1055	0.0591	0.0577	0.0051

Table 2
Comparison of terminal estimation errors.

	$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_4$	$\mathcal{L}_5$	$\mathcal{L}_6$
D-Nash ($g = 10$)	0.066	0.044	0.042	0.049	0.055	0.042
SH-Nash ($g = 10$)	0.059	0.040	0.037	0.047	0.049	0.036

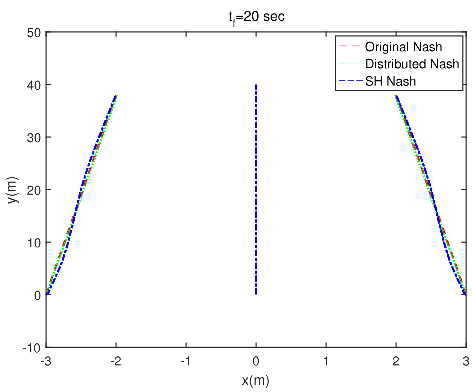

Fig. 2. Trajectories under different control strategies over $\mathcal{L}_1$.

which means their initial positions are equally spaced and agent 1's initial velocity is upward and agents 2 and 3's initial velocities are equal to zero. We assume that the desired formation among the agents is a triangle with the displacement vectors given by

$$\mu_{12} = [2\ \ 2]^T, \quad \mu_{13} = [-2\ \ 2]^T, \quad \mu_{23} = [-4\ \ 0]^T,$$

In performance indices (2), $F_f = F_o = R = I$, $r_i = f_{ij} = 1, f_i = 0.01$, $\forall i, j$ and $t_f = 20$ second for the time interval of the game. For comparison purpose, we first solve (40) with open-loop Nash strategy defined in (9) (i.e., denoted by Original Nash hereafter). Then, we proceed to implement the distributed open-loop Nash strategy defined in (16) with $g_i = 10$ chosen in (15) (i.e., denoted by D-Nash hereafter). Consequently, we solve (40) with shrinking horizon Nash strategy defined in Theorem 4 with $g_i = 10$ and 5 sampling instances (i.e., denoted by SH-Nash hereafter).

It follows from Fig. 2 that the trajectories generated by all three schemes are essentially identical, while, as expected, the shrinking horizon Nash strategy exhibits a better transient performance.

The terminal estimation errors are summarized in Table 2, and it is apparent that both strategies achieve acceptable performance. Furthermore, in order to quantify performance of the proposed protocols, we define the total terminal formation error with respect to the desired displacement as follows

$$\Omega = \frac{1}{2}\sum_{i=1, j\neq i}^{3} \|p_i(t_f) - p_j(t_f) - \mu_{ij}\|, \tag{41}$$

As such, the total terminal formation errors based on different Nash strategies and g over $\mathcal{L}_1$ to $\mathcal{L}_6$ can be summarized into Table 1, which demonstrates that the original Nash strategy exhibits a better, almost perfect, performance in all scenarios, while the shrinking horizon strategy performs better than distributed Nash strategy, and the total formation error decreases with a larger g, as expected, a larger gain will in general lead to a more accurate estimate on terminal state, this verifies the conjecture of the proposed optimization strategy.

7. Conclusion

This paper considers the design of open-loop Nash strategy for multi-agent formation control problems. The proposed approach utilizes the distributed estimation of the terminal state variables through network-based information exchange. The convergence rate of the terminal state estimation algorithm can be made exponential with proper choice of the gain and the proposed distributive Nash strategy can guarantee an $\epsilon-$Nash equilibrium. An illustrative example is proposed to examine performance of the proposed scheme. Future work on this subject should focus on enabling closed-loop Nash strategy among multi-agent systems.

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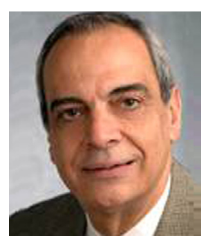

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