Fostering Wireless Spectrum Sharing via Subsidization

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***Abstract*— Allowing an improved and ubiquitous sharing of the precious radio spectrum among providers has been identified as a key goal by many governmental agencies. In this paper, we propose a novel approach to foster more sharing of the radio spectrum via the use of regulatory power. We develop a spectrum subsidization scheme in which providers are incentivized by cheaper spectrum allocation fees from the regulators in return for "proof-of-sharing". The providers are offered discounted spectrum bands, potentially at different locations, but will be asked to provide coverage to users that are not subscribed to them so as to maintain their subsidy incentives from the government. We introduce a game-theoretic framework to analyze the advantages and drawbacks of such a subsidy scheme and explore potential regulatory guidelines.**

***Index Terms*— spectrum sharing; subsidy markets; game theory**

I. Introduction

With the continuous increase in mobile traffic, users and mobile communication services are facing the problem of spectrum shortages. An 18-fold increase in mobile traffic is expected by 2016 [1]. Governments around the world are trying to find ways in which more spectrum can be made available not only for mobile use, but also for other services that involve the use of wireless broadband technologies such as weather forecast and surveillance [2]. Moreover, wireless networking technologies are becoming a more critical platform for disaster management and public safety applications throughout the world, which mandates better communications and interoperability following a disaster [3].

Developing an efficient spectrum policy and new regulations regarding spectrum use are essential in this day and age. Every three years, representatives and delegates from many nations get together to discuss the future of spectrum policies and ways to make spectrum use more efficient and equitable. The World Radio communication Conference (WRC), supported by the United Nations, is usually held in the International Telecommunication Union (ITU) head quartered in Geneva [4]. The US plays an important role nationally and internationally in steering and influencing the global spectrum policy and its national policies on spectrum use and regulation.

The main issue and challenge ahead is to create additional spectrum capacity. Regulators have three policy directions to increase the spectrum capacity: (i) increasing availability and access to radio spectrum for wireless broadband via allocating additional spectrum; (ii) reassigning spectrum to new users; and (iii) opening up spectrum for unlicensed use. Other policy options that may be employed to increase spectrum capacity include requiring that wireless network infrastructure be shared; changing the cost structure of spectrum access; moving to more spectrum-efficient technologies; and sharing spectrum. The US government is currently supporting innovative approaches to spectrum sharing. There is a new Department of Defense (DoD) Initiative that is funding research and proposals for spectrum sharing and increasing the capacity of wireless broadband services in the country. The FCC and NTIA are investing in spectrum sharing projects as means to increase spectrum capacity while increasing its efficiency as well. For example, T-Mobile USA is sharing its spectrum with federal agencies. Similarly, the NTIA is also following a plan and timetable for making its spectrum available for commercial use.

A major impediment for pervasive spectrum sharing is the providers' tendency to protect the bands they earned with a lot of licensing and operating costs. Adopting new technologies to facilitate device-to-device (D2D) spectrum sharing is becoming an important policy consideration for spectrum management. Current spectrum usage heavily follows competitive auctions [5] which are healthy for balancing standardization trends [6], but may hinder the potential gains that can be obtained via extensive sharing [7], [8]. Current broadband policies [9] dictate that competitive auctions must remain intact, while simultaneously incorporating elements of spectrum sharing and new ways to manage spectrum usage (e.g., via D2D). In such policies, developing a governance structure for public safety broadband networks and making more spectrum available for public safety is critical [5]. Such governance and policies welcoming the necessary economic tools for enabling heterogeneous spectrum sharing for *the*

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larger good are imperative. Recently, agencies such as DARPA, FCC, and NSF are soliciting projects that develop innovative ways for spectrum sharing [2]. Meanwhile, AT&T, Verizon and T-Mobile have signed a deal to test spectrum sharing with the DoD. This will help determine if federal spectrum use will be adversely affected when commercial services are in the bank as well as how the DoD's sharing of the spectrum will impact commercial mobile services [10].

One promising approach to foster more sharing of the spectrum over large-scale D2D spectrum sharing markets is via the use of regulatory power. In fact, the National Broadband Plan [9] recommends the wide-spread development of the concept of "spectrum subsidy", e.g., licensing of the D block for commercial use if *public safety partnerships* are considered by the licensee. In this paper, we investigate this idea of subsidizing the spectrum to the providers with lower costs in return of "proof-of-sharing". Thus, the providers will be offered discounted spectrum bands, potentially at different locations, but will be asked to cover users not subscribed to them so as to maintain their subsidy incentives from the government (i.e., to sustain spectrum sharing techniques via D2D). Recent studies suggest significant market and user welfare gains under such subsidization, e.g., data subsidy for offering minimal data plan to users for free [11] or spectrum pooling among providers to improve user experience [7]. However, a game-theoretic framework to analyze a subsidized spectrum market, that considers spatial and temporal provider-government relationships as well as dynamic components like D2D sharing, roaming, and signal quality, is essentially unexplored.

The rest of the paper is organized as follows. Section II discusses related work on spectrum sharing via subsidization. A spectrum market between government, providers, and customers is presented in Section III, and some key observations are highlighted. Section IV concludes the paper, followed by an appendix on a two-provider two-region case of the proposed spectrum market.

II. Related Work

Regulatory practice has been part of the wireless market since its inception as the resource being shared was soon recognized to be very valuable. Governmental agencies establish regulatory policies to increase the efficiency of the spectrum market while preserving an acceptable level of fairness in sharing the precious spectrum. The concept of subsidization is one way to involve the government so as to incentivize various long-term goals in the market being regulated. Typically, subsidy regulations are used for markets serving "the larger good" (i.e., the benefit of the whole society) and involving a significant amount of cost producers cannot bare individually. Transportation [12], agriculture [13] and telecommunications [14] are examples of such markets where new initiatives require significant infrastructure investments, and hence the government's involvement is justified. However, subsidization is known to have negative effects on the efficiency of the market. Thus, performance based subsidization is preferred if at all possible.

Even though subsidization is heavily employed in several markets, the wireless market has not seen much of its usage beyond a few, limited scenarios. The most known subsidization in a wireless market is the subsidization of the expensive phones to the users by the provider [15], in that the user pays the phone's cost over a locked, termed contract. Unlocking the contract term either requires return of the phone or payment of a significant fee.

Only recently, Yu and Kim [11] focused on the concept of using subsidies for spectrum management. Their work analyzed price and quality of service (QoS) subsidy schemes to increase the utility of the consumers from the data plans. The goal is to increase the availability of wireless data plans to more users. In their model, the regulator offers a subsidized (i.e., cheaper) data plan to the end users with lower quality via subsidizing to the providers. In our work, we consider subsidizing the spectrum to the providers and focus on implementing a subsidy system mainly between the providers and the government. We aim to make the subsidization seamless to the end users and aim to incentivize the providers to be more welcoming to the users subscribed to other providers. As a major difference, we focus on spectrum subsidy rather than data plan subsidy.

A related concept to the subsidization approach considered in this paper is inter-operator *roaming*. Through roaming, a customer can make/receive calls and send/receive data while outside of the home network coverage, by using the infrastructure of another provider. This is enabled through reciprocal roaming charges among operators, which may vary e.g. depending on the nationwide coverage supported by an operator through its infrastructure. The new roaming regulations in European Union (EU) to be effective from July 1st, 2014, will allow end users to choose providers for international roaming services, while they are within the EU [16]. The goal is to facilitate competition in the roaming market and bring international roaming prices down to domestic rates, e.g., through diversity of market players, low barriers to market entry, and equal access to basic wholesale services. The analysis in [17] shows that, as long as certain coverage conditions are fulfilled, providers in an open system (where roaming is allowed) have strictly better revenues when compared to a closed system with no roaming. Benefits of inter-operator spectrum sharing and joint radio resource management (JRRM) have also been demonstrated in recent works [18], [19], [20]. The inter-operator JRRM technique proposed in [18] allows subscribers to get service through other operators in case the home operator network is blocked, and results show that inter-operator roaming agreements improve both the network performance and operators' revenue.

In [19], a market model is introduced, which uses the roaming rate as an incentive for service providers to gain revenue, when they allow other users to access their network. Optimal roaming rate is derived to maximize the social welfare of all the service providers and licensed users. Spectrum sharing agreements among operators, while providing operators an alternative means of meeting their bandwidth demands, may also encourage them to under-invest in their

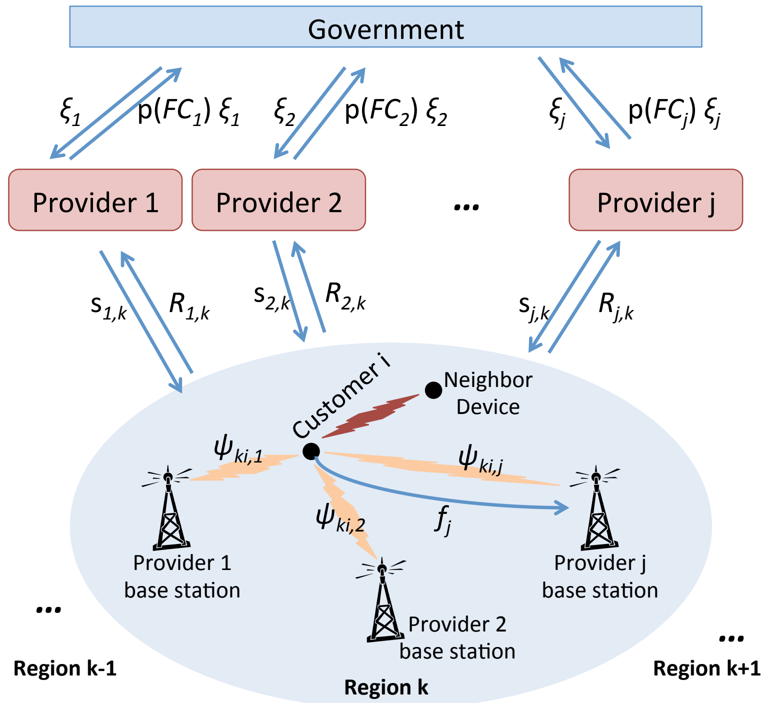

Fig. 1. Model for a subsidized spectrum market.

infrastructure. Such tradeoffs have been investigated in [20] considering two different sharing models. Our approach for subsidization, while also allows inter-operator sharing of spectrum similar to roaming, is fundamentally different than these earlier work, since it involves the government as the subsidy source to facilitate spectrum sharing.

III. A Spectrum Market with Subsidy

Even though there is a strong literature on the role of government in market formations [21], [22], the government's regulatory role in the radio spectrum market has not been explored well within a game-theoretic framework. We propose a spectrum market model with three types of agents as shown in Fig. 1: customers, providers, and a single government player. The customers are essentially the end-user devices that will ultimately engage in localized spectrum sharing markets. Let $\mathcal{I} = \{1, ..., I\}$ be the set of wireless customers. These customers are spread out over a set of regions $\mathcal{K} = \{1, ..., K\}$. We assume there are n_k customers in region k, $k = 1, ..., K$. If k_i is the region in which customer i is located, we call k_i her "home region". Each customer i makes β_i calls in her home region (over the time period associated with the game); for simplicity we assume β_i is equal to a constant β for all i. In addition, we will assume the customers each make α calls outside their home region.

To choose her main service provider, customer i takes as given the "intensity of service/signal" $\psi_{k_i j}$ offered by each provider j in region k_i, as well as the fee f_j that each j charges for service. This is done probabilistically as described below.

The set of all providers is denoted by $\mathcal{J} = \{1, ..., J\}$. Providers operate in all regions. The government gives each provider an allotment of bandwidth and of monetary subsidy, which they, in turn, allocate for their own use in each of the K regions. The amount of bandwidth and money spent in each region determines the intensity of service offered, which in turn helps determine who wins customer subscriptions. An important feature of this model is that the government motivates the providers to give service to customers who are outside of their home region (i.e., roaming) or simply far away from the base stations of their "home providers". "Foreign" providers might be able to provide a higher intensity signal than the customers' home providers. The government aims to motivate foreign providers by reducing each provider's subsidy if the provider does not service enough number of "foreign" customers who are away from their home providers.

We propose a two-stage, extensive form noncooperative game in which the government moves first, followed by the providers (all simultaneously). Here, the customers are *not* formal players in the game because their actions are completely determined by the actions of the other players. Mathematically, we could solve for a perfect Nash equilibrium in the game as follows: a) Knowing what the customers will do as a function of $\{\psi_{k_i j}\}_{i,j}$ and $\{f_j\}_{j=1}^{J}$, we can solve for the optimal strategy for the providers; b) Knowing how the providers/customers behave, we can solve the government's problem, which would be to maximize social welfare – which in this case would be the total of customers' utility from consuming the wireless spectrum.

A. Customer i's Problem

Each customer i has the same increasing concave utility function u for intensity of wireless service, expressed in units of money per call. Thus

$$U_j(k_i) = \beta u(\psi_{k_i j}) - f_j \tag{1}$$

measures the utility she would get from local calls in her home region k_i, if she chose provider j as her "home provider". She then chooses provider j ($j = 1, ..., J$) with probability

$$P_j(k_i) = \frac{U_j(k_i)}{\sum_{j=1}^{J} U_j(k_i)}. \tag{2}$$

This probabilistic assignment of contracts (prizes) to providers (contestants) based on resources offered is taken from "contest theory", and arises in many applications (see, e.g., [23], [24], or the excellent survey paper of [25]).

We note that the customers choose the providers based only upon offered service in their "home regions", because their "outside calls" will always be covered (at no extra cost to them) as a result of the government's subsidization scheme.

B. Provider j's Problem

From the above, we see that provider j will capture $n_k \cdot P_j(k)$ of the customers in region k, for a revenue of

$$R_{jk} = f_j \cdot n_k \cdot P_j(k). \tag{3}$$

Besides $\sum_{k=1}^{K} R_{jk}$, provider j also receives a subsidy of ξ_j from the government, s_{jk} of which it designates/spends to region k, $k = 1, ..., K$. In addition, it is given B_j in bandwidth, of which it allocates b_{jk} to region k. Then the intensity of service that it offers in region k is

$$\psi_{kj} = Q(s_{jk}, b_{jk}), \tag{4}$$

where Q is an exogenously given function (increasing and concave).

Next, we note that the customers in Region k^* make $\alpha \cdot n_{k^*}$ calls outside of their home region. We assume

$$n_{k^* \hat{k}} = \frac{\alpha \cdot n_{k^*} \cdot n_{\hat{k}}}{\sum_{k \neq k^*} n_k} \tag{5}$$

of these occur in region $\hat{k}$ ($\hat{k} \neq k^*$). These calls will be allocated probabilistically to the providers, according to the intensity levels offered by the providers in region $\hat{k}$. The total number of such "outside calls" that provider j serves is then

$$\mathcal{FC}_j = \sum_{k=1}^{K} \sum_{k^* \neq k} n_{k^* k} \frac{u(\psi_{kj})}{\sum_{j=1}^{J} u(\psi_{kj})}. \tag{6}$$

The provider is then assessed a penalty according to this number – the lower the quantity $\mathcal{FC}_j$ is, the higher the penalty. The penalty is expressed as a proportion of subsidy lost, and is incurred after the market. So, assuming this penalty function is denoted $p(x)$, we can write down an optimization problem for provider j:

$$\max_{\{s_{jk}\},\{b_{jk}\},f_j} \sum_{k=1}^{K} R_{jk} + (1 - p(\mathcal{FC}_j))\xi_j - \sum_{k=1}^{K} s_{jk} \tag{7}$$

such that

$$\sum_{k=1}^{K} s_{jk} \leq \xi_j, \tag{8}$$

$$\sum_{k=1}^{K} b_{jk} = B_J, \tag{9}$$

$$\psi_{kj} = Q(s_{jk}, b_{jk}), \tag{10}$$

$$s_{jk}, b_{jk}, f_j \geq 0 \ \forall \ j, k. \tag{11}$$

where the first term in the maximization is the total revenue, the second term is the leftover subsidy money after the penalty is deducted due to fewer-than-required sharing of the provider's spectrum, and the last term is the total amount of expenses the provider uses from the subsidy it received from the government.

C. Government's Problem

Finally, the government's decision variables are ξ_j and B_j, $j = 1, ..., J$. It also can designate the function $p(x)$ described above. Its objective is to maximize the customers' total utility from wireless service, which is

$$\max_{\{\xi_j, B_j\}_{j=1}^{J}} \Bigg[\sum_{i=1}^{I} \beta u(\psi_{k_i j(i)}) + \sum_{j=1}^{J} \sum_{k=1}^{K} \Big(\sum_{k^* \neq k} n_{k^* k} \frac{u(\psi_{kj})}{\sum_{j=1}^{J} u(\psi_{kj})} u(\psi_{kj}) \Big) \Bigg] \tag{12}$$

such that

$$\sum_{j=1}^{J} B_j = B \tag{13}$$

$$\sum_{j=1}^{j} \xi_j \leq \xi \tag{14}$$

$$B_j, \xi_j \geq 0 \ \forall j. \tag{15}$$

Here, the first term is the utility that customers acquire in their home regions, while the second term is from calls in outside regions. The notation $j(i)$ is the "home" provider to which customer i is assigned. The quantities B and ξ represent the total amounts of bandwidth and subsidy that the government has at its disposal to allocate. Due to the complexity of the above model, fully analytical solutions might be challenging but will be attempted. On the other hand, computation will be used to analyze various strategies that the government and providers might consider.

D. Observations and Discussion

The model we have just outlined, while not perfectly covering all details and aspects, provides a good framework to consider the viability of subsidizing spectrum to providers in return for increased spectrum sharing. The providers constitute a key player to scrutinize in such a spectrum market model. Ultimately, the goal of this market model is to motivate the providers to share their spectrum among each other to attain the larger good of serving more customers with better quality access links.

TABLE I
NOTATIONS AND SYMBOLS

Symbol	*Explanation*
n_k	Number of customers in region k
k_i	The region where customer i is located
f_j	The fee charged by provider j to its customers
ψ_{kj}	The intensity/quality of provider j's signal in region k
$Q(.)$	An increasing concave function – parameterized with the investment, s_{jk}, and/or bandwidth, b_{jk}, in a region
$u(.)$	Customers' utility function – parameterized with received signal intensity ψ_{kj}
$U_j(k)$	The utility obtained by choosing provider j in region k
$P_j(k)$	The probability that provider j is chosen in region k
R_{jk}	Provider j's revenue collected from region k
ξ_j	Subsidy amount provider j receives from the government
s_{jk}	Portion of the subsidy amount provider j designates/spends in region k
ξ	Total subsidy budget of the government
β	Number of calls made by a customer in her home region(s)
α	Number of outside/foreign calls a customer makes
$\mathcal{FC}_j$	Number of foreign calls provider j serves
$p(.)$	Penalty function determining the proportion of the subsidy to be returned to the government – parameterized with $\mathcal{FC}_j$

Provider j's problem (7) offers several insights. The *first term* in (7) is the fees provider j collects from its home customers, who are subscribed to provider j. Provider j has to serve these customers by default. Customers in their current location, however, may be able to find another provider that offers a better quality signal at that location. In that case, those customers would be considered as making a foreign call, and thus contributing to the sharing of the spectrum among the providers. Further, the revenue from the first term may be coming from cellular or device-to-device (D2D) subscriptions.

The *second term* in (7) describes the amount of subsidy money left to provider j. This leftover amount is dependent on the subsidy amount, ξ_j, given by the government to provider j, and more importantly, the number of foreign calls, $\mathcal{FC}_j$, served by provider j during the subsidy term. If $\mathcal{FC}_j$ is not large enough, provider j may have to return all of the subsidy back to the government. At the other extreme, provider j may keep all of the subsidy amount, ξ_j, if it did serve sufficient number of foreign customers.

The *third term* in (7) quantifies the sum of investments provider j makes from its subsidy. Provider j can choose which regions to invest in more. It can further decide whether to spend all of the subsidy it received, ξ_j, from the government. In this model, the provider takes some risk by spending the subsidy money. It needs to wisely decide on which regions to invest. The ideal situation for the provider is to invest into those regions in which more foreign calls become possible to serve. This will motivate the providers to invest into more congested areas and thus serve the larger goal of improving user-perceived link quality.

Observation 1: *Providers are unlikely to get hurt by the subsidization option.*

Inspecting (7), the provider does not have to take the subsidization option if it will not generate any profits. The second and third terms of (7) will evaluate to zero in such a case, and the revenue from the first term will still be there.

Observation 2: *Providers can easily be motivated into a subsidized market.*

The government can offer a large ξ_j and a conservative (i.e., not heavily penalizing) penalty function to attract the providers into the subsidization option. Since, from Observation 1, there is really not much of risk from the subsidization option, the tipping point for providers to sign into subsidization contracts will not be high. A relatively high ξ_j will promise a positive return from the subsidization.

Observation 3: *Providers will be motivated to invest in a non-overlapping manner and collectively cover a larger area.*

The number of foreign calls $\mathcal{FC}_j$ in (7) depends on the amount of overlap between the network coverage areas of different providers. Operators, by not duplicating their infrastructure in the same areas, can minimize their investment cost s_{jk}, while at the same time minimizing the penalty cost $(1 - p(\mathcal{FC}_j))\xi_j$ charged by the government in (7), yielding high revenues. This observation is aligned with the conclusions in [17], in which the analysis suggests that minimization of the duplication of network infrastructure by different providers yields higher provider revenues.

Observation 4: *As $\frac{\alpha}{\beta} \to \infty$, subsidization will be more beneficial, particularly to those providers smaller in size.*

It follows from (5) and (6) that the number of foreign calls served by provider j, $\mathcal{FC}_j$, will increase by α. This will in turn increase the contribution of the second term in (7) to provider j's revenue. An interesting situation happens for smaller providers. For those providers whose customers are making more foreign calls than home calls, the contribution of the second and third term of (7) into their revenue total will be more than the first term's contribution.

Observation 5: *Small providers will be able to compete with large providers by keeping $\alpha \geq \beta$.*

This follows from Observation 4. A new note to make here is that small providers with smaller infrastructure will be able to compete against providers with large infrastructure as long as they maintain the balance $\alpha \geq \beta$. They will be able to do so by swiftly investing into congested spots where larger providers cannot reach well. Further, D2D sharing will make more sense for those providers with small or no infrastructure but can offer seamless sharing capability among devices.

Observation 6: *Monopolization will be avoided as long as providers are attracted to the subsidization market.*

This follows from Observations 4 and 5. A monopolizing provider will have $\frac{\alpha}{\beta}$ ratio closer to 0. That will mean that subsidization will not be so beneficial for him. Since it will be possible to receive and retain subsidy from the government, a smaller provider will be able to attain similar revenues by keeping $\alpha \geq \beta$. Thus, there will be no incentive to monopolize.

IV. Summary and Future Work

We have proposed a spectrum market where sharing is promoted explicitly by the government. The government offers subsidy support to the wireless operators and requires a performance metric to be reported. We considered this metric to be “the number of foreign customers” served by the operator, where “foreign customer” means a customer who is not subscribed to that particular operator. This way, the operators are motivated to invest their subsidy support into regions where likelihood of reaching and serving foreign customers is higher. This, in turn, can increase the aggregate coverage area and the overall signal quality the customers receive. Both of these end results are appealing to the government and are the main reasons why the government should subsidy.

Our approach is different from the typical subsidization with open ended lending of the money to the providers. We detailed a model capturing the essence of the model with simplifying assumptions. We laid out the game theoretic framework between the government and the providers. We derived the first-order conditions for a Nash equilibrium. We further specialized the general problem for a two-provider and two-region scenario.

We made several observations indicating various benefits of a spectrum market with performance-based subsidy. We qualitatively showed that such a spectrum market will be attractive to the operators and discourage monopolization. Further, we observed that small providers with relatively smaller infrastructure can possibly survive in this type of market and successfully compete against large operators.

Possible future work includes extending the model to finer granularity abstractions. For instance, considering different utility functions for each customer may provide better insights and guidance for providers and the government for policy making. Likewise, consideration of heterogeneous provider profiles, e.g., each serving a different set of regions, may help in the same way as well. Further, it will be interesting to investigate various special and maybe extreme cases of this market model such as consideration of the viability of D2D providers with no infrastructure in this kind of market. Lastly, co-existence of this sort of spectrum sharing market with the traditional “closed” spectrum market with minimal sharing is another issue to evaluate in order to understand dynamics of transitioning into a “pervasive” sharing market.

References

[1] “Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 20122017 (White Paper),” Cisco Systems, February 2012.

[2] T. Power and L. Strickling, “Supporting innovative approaches to spectrum sharing,” Office of Science and Technology Policy website, 2013.

[3] A. M. Mousa, “Challenges of future r&d in mobile communications,” *International Journal of Advanced Computer Science and Applications*, vol. 3, no. 10, pp. 1–10, 2012.

[4] A. W. Clegg, “Results from the 2012 world radio communication conference [spectrum policy and regulatory issues],” *IEEE Wireless Communications*, vol. 19, no. 3, pp. 6–7, 2012.

[5] L. K. Moore, “Spectrum policy in the age of broadband: Issues for congress,” CRS Report for Congress, Feb. 2010.

[6] R. I. Desourdis (Editor), “Emerging public safety wireless communication systems,” Artech House Mobile Communication Series, 2002.

[7] N. Jesuale, “Lights and sirens broadband – how spectrum pooling, cognitive radio, and dynamic prioritization modeling can empower emergency communications, restore sanity and save billions,” in *Proceedings of IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, May 2011, pp. 467–475.

[8] William J. Baumol and Dorothy L. Robyn, *Toward an Evolutionary Regime for Spectrum Governance: Licensing or Unrestricted Entry?*, Brookings Institution Press, February 2006.

[9] “Connecting america: The national broadband plan,” FCC, 2010.

[10] P. Goldstein, “AT & T, Verizon, T-Mobile forge pact to explore spectrum sharing with government,” FierceWireless website, Jan. 2013.

[11] Seung Min Yu and Seong-Lyun Kim, “Guaranteeing user welfare in network service: Comparison of two subsidy schemes,” *Performance Evaluation Review*, vol. 40, no. 2, pp. 22–25, September 2012.

[12] MatthewG. Karlaftis and PatrickS. McCarthy, “Subsidy and public transit performance: A factor analytic approach,” *Transportation*, vol. 24, no. 3, pp. 253–270, 1997.

[13] Shigemi Yabuuchi, “Urban unemployment, international capital mobility and development policy,” *Journal of Development Economics*, vol. 41, no. 2, pp. 399 – 403, 1993.

[14] SteveG. Parsons, “Cross-subsidization in telecommunications,” *Journal of Regulatory Economics*, vol. 13, no. 2, pp. 157–182, 1998.

[15] Uday Kelkar and Naveen Aerrabotu, “Over-the-air subsidy lock resolution,” USPTO Patent US 7088988 B2, 2006.

[16] “Roaming regulation – choice of decoupling method,” Tech. Rep. BoR (12) 68, Board of European Regulators for Electronic Communications (BEREC), June 2012.

[17] Simona Fabrizi and Bruno Wertlen, “Roaming in the mobile internet,” *Telecommunications Policy*, vol. 32, no. 1, pp. 50–61, 2008.

[18] L. Giupponi, R. Agusti, J. Perez-Romero, and O. Sallent, “Improved revenue and radio resource usage through inter-operator joint radio resource management,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2007, pp. 5793–5800.

[19] Hung-Bin Chang and Kwang-Cheng Chen, “Cooperative spectrum sharing economy for heterogeneous wireless networks,” in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM) Workshops*, 2011, pp. 458–463.

[20] R. Berry, M. Honig, T. Nguyen, V. Subramanian, H. Zhou, and R. Vohra, “On the nature of revenue-sharing contracts to incentivize spectrum-sharing,” in *Proc. IEEE INFOCOM*, 2013, pp. 845–853.

[21] M. Shubik, *The Theory of Money and Financial Institutions*, vol. 3, MIT Press, Cambridge, MA, 2011.

[22] Thomas Quint and Martin Shubik, *Barley, Gold, and Fiat: A Pure Theory of Money (in press)*, Yale University Press, 2013.

[23] S. Chowdhury, D. Kovenock, and R Sheremeta, “An experimental investigation of colonel blotto games,” *Economic Theory*, vol. 52, pp. 833–861, 2013.

[24] D. Kovenock and B. Roberson, chapter Conflicts with Multiple Battlefields, pp. 503–531, Oxford Handbook of the Economics of Peace and Conflict. Oxford University Press, New York, 2010.

[25] E. Dechenau, D. Kovenock, and R. Sheremeta, “A survey of experimental research on contests, all-pay auctions, and tournaments,” Chapman University, Economic Science Institute, working paper number 12-22, 2012.

Appendix I
A Simplified Market for Spectrum Subsidy

We attempt to simplify the market model in Section III so that initial observations become possible. In particular, we make the following two simplifications:

- Eliminating explicit handling of bandwidth, assuming it is implicit in the intensity function expressing received signal quality on an access link. Hence, $\psi_{kj} = Q(s_{kj})$.
- We assume a linear penalty function $p(\mathcal{FC}_j)$. This implies that the $(1 - p(\mathcal{FC}_j))\xi_j$ term in (3) can be recast as (a constant plus) $\delta\mathcal{FC}_j$, where δ is a per-foreign-customer reward that the government gives to providers.

By making these simplifying assumptions and substituting the expression (3) for R_{jk} into (7), we may rewrite the provider j's problem as follows:

$$\max_{\{s_{j^*k}\}_{k=1}^K, f_{j^*}} \quad \sum_{k=1}^K f_{j^*} n_k \frac{\beta u(Q(s_{j^*k})) - f_{j^*}}{\sum_{j=1}^J \beta u(Q(s_{jk})) - f_j} + \delta \sum_{k=1}^K \sum_{\hat{k}\neq k} n_{\hat{k}k} \frac{u(Q(s_{j^*k}))}{\sum_{j=1}^J u(Q(s_{jk}))} - \sum_{k=1}^K s_{j^*k} \quad (16)$$

such that

$$\sum_{k=1}^K s_{j^*k} \leq \xi_j, \quad (17)$$

$$s_{j^*k} \geq 0, k = 1...K, \quad (18)$$

$$f_{j^*} \geq 0, \quad (19)$$

where δ is specified by the government for penalizing providers not serving sufficient number of foreign customers. Economically, δ is the per foreign customer reward that the government gives to providers.

In order to have a perfect Nash equilibrium, each provider must be optimizing her own problem. Hence, we take the first-order conditions for the system (16)-(19) to obtain the first-order conditions. We let $\sqcap(\cdot) = u(Q(\cdot))$ to simplify the notation, and write the first-order conditions as follows:

1) w.r.t. f_{j^*}:

$$\sum_{k=1}^K n_k \left[(\beta\sqcap(s_{j^*k}) - 2f_{j^*}) \sum_{j=1}^J [\beta\sqcap(s_{jk}) - f_j] + f_{j^*} (\beta\sqcap(s_{j^*k}) - f_{j^*}) \right] = 0 \quad (20)$$

2) w.r.t. $s_{j^*k^*}$:

$$\frac{f_{j^*} n_{k^*} \beta\sqcap'(s_{j^*k^*}) \sum_{j=1}^J [\beta\sqcap(s_{jk^*}) - f_j] - (\beta\sqcap(s_{j^*k^*}) - f_{j^*}) \beta\sqcap'(s_{j^*k^*})}{\left(\sum_{j=1}^J [\beta\sqcap(s_{jk^*}) - f_j]\right)^2} + \delta \sum_{k\neq k^*} n_{k^*k} \sqcap'(s_{j^*k^*}) \frac{\left(\sum_{j=1}^J \sqcap(s_{jk^*})\right) - \sqcap(s_{j^*k^*})}{\left(\sum_{j=1}^J \sqcap(s_{jk^*})\right)^2} - 1 - \lambda_j = 0. \quad (21)$$

3)

$$\lambda_j \geq 0, \sum_{k=1}^K s_{j^*k} = \xi_j \quad \text{or} \quad \lambda_j = 0, \sum_{k=1}^K s_{j^*k} \leq \xi_j. \quad (22)$$

4)

$$f_{j^*} \geq 0, s_{j^*k} \geq 0, k = 1...K. \quad (23)$$

Here, λ_j is the Lagrangian multiplier for the constraint (17) on the total amount of subsidization the government offers to providers. We remind the reader that the previous only finds the optimal strategies for the providers as a function of the government's ξ and δ. As part of the perfect Nash equilibrium we would then have to solve the government's problem (with its decision variables being ξ and δ) given it knows the providers' response functions to ξ and δ.

APPENDIX II
CASE WITH TWO REGIONS AND TWO PROVIDERS

To make our model and equilibrium solutions more understandable, we now rewrite the providers' problem and derive the first-order conditions for the case with two providers and two regions. We further make the following simplifying assumptions:

- Concave customer utility function of the form: $u(x) = 2\sqrt{x}$.
- A linear signal quality function: $Q(s_{jk}) = s_{jk}$.

In general, a concave utility is considered realistic enough to capture the diminishing returns behavior of received quality. So, the former assumption is fine. The latter assumption may be too optimistic since signal quality will also behave in a diminishing returns manner as the investment on the infrastructure increases. However, since $Q(\cdot)$ always feeds into $u(\cdot)$ in our formulation, the $u(Q(\cdot))$ will still behave according to diminishing returns.

A. Provider 1's Problem

Given ξ_1 and δ, Provider 1 will have to optimize w.r.t. s_{11}, s_{12}, and f_1. Thus, its problem will be as follows:

$$\max_{s_{11}, s_{12}, f_1} \quad f_1 n_1 \frac{2\beta\sqrt{s_{11}} - f_1}{2\beta\sqrt{s_{11}} - f_1 + 2\beta\sqrt{s_{21}} - f_2} + f_1 n_2 \frac{2\beta\sqrt{s_{12}} - f_1}{2\beta\sqrt{s_{12}} - f_1 + 2\beta\sqrt{s_{22}} - f_2} + \delta \left(n_{21} \frac{\sqrt{s_{12}}}{\sqrt{s_{12}} + \sqrt{s_{22}}} + n_{12} \frac{\sqrt{s_{11}}}{\sqrt{s_{21}} + \sqrt{s_{11}}} \right) - s_{11} - s_{12} \quad (24)$$

such that

$$s_{11} + s_{12} \leq \xi_1, \quad (25)$$

$$s_{11}, s_{12}, f_1 \geq 0. \quad (26)$$

The first-order conditions on the above problem will then be:

1) w.r.t. f_1:

$$2\, n_1 (\beta\sqrt{s_{11}} - f_1)(2\beta\sqrt{s_{11}} - f_1 + 2\beta\sqrt{s_{21}} - f_2) + n_1 f_1 (2\beta\sqrt{s_{11}} - f_1) + 2n_2 (\beta\sqrt{s_{12}} - f_1)(2\beta\sqrt{s_{22}} - f_2 + 2\beta\sqrt{s_{12}} - f_1) + n_2 f_1 (2\beta\sqrt{s_{12}} - f_1) = 0 \quad (27)$$

2a) w.r.t. s_{11}:

$$\frac{\beta f_1 n_1 (2\beta\sqrt{s_{21}} - f_2)}{\sqrt{s_{11}} (2\beta\sqrt{s_{11}} - f_1 + 2\beta\sqrt{s_{21}} - f_2)^2} + \frac{\delta n_{12}\sqrt{s_{21}}}{2\left(\sqrt{s_{21}} + \sqrt{s_{11}}\right)^2 \sqrt{s_{11}}} - 1 - \lambda_1 = 0 \quad (28)$$

2b) w.r.t. s_{12}:

$$\frac{\beta f_1 n_2 \left(2\beta\sqrt{s_{22}} - f_2\right)}{\sqrt{s_{12}}\left(2\beta\sqrt{s_{12}} - f_1 + 2\beta\sqrt{s_{22}} - f_2\right)^2} + \frac{\delta n_{21}\sqrt{s_{22}}}{2\left(\sqrt{s_{12}} + \sqrt{s_{22}}\right)^2\sqrt{s_{12}}} - 1 - \lambda_1 = 0 \quad (29)$$

3)

$$\lambda_1 \geq 0,\ s_{11} + s_{12} = \xi_1 \quad \text{or} \quad \lambda_1 = 0,\ s_{11} + s_{12} \leq \xi_1 \quad (30)$$

4)

$$f_1 \geq 0,\ s_{11} \geq 0,\ s_{12} \geq 0.$$

B. Provider 2's Problem

Similar to Provider 1, given ξ_2 and δ, Provider 2 will have to optimize w.r.t. s_{21}, s_{22}, and f_2. So, the first-order conditions for Provider 2 will then be:
1) w.r.t. f_2:

$$\begin{aligned} &2\ n_1 \left(\beta\sqrt{s_{21}} - f_2\right)\left(2\beta\sqrt{s_{11}} - f_1 + 2\beta\sqrt{s_{21}} - f_2\right) \\ &\quad + n_1 f_2 \left(2\beta\sqrt{s_{21}} - f_2\right) \\ &\quad + 2n_2 \left(\beta\sqrt{s_{22}} - f_2\right)\left(2\beta\sqrt{s_{12}} - f_1 + 2\beta\sqrt{s_{22}} - f_2\right) \\ &\quad + n_2 f_2 \left(2\beta\sqrt{s_{22}} - f_2\right) = 0 \qquad (31) \end{aligned}$$

2a) w.r.t. s_{21}:

$$\frac{\beta f_2 n_1 \left(2\beta\sqrt{s_{11}} - f_1\right)}{\sqrt{s_{21}}\left(2\beta\sqrt{s_{11}} - f_1 + 2\beta\sqrt{s_{21}} - f_2\right)^2} + \frac{\delta n_{12}\sqrt{s_{11}}}{2\left(\sqrt{s_{21}} + \sqrt{s_{11}}\right)^2\sqrt{s_{21}}} - 1 - \lambda_2 = 0 \quad (32)$$

2b) w.r.t. s_{22}:

$$\frac{\beta f_2 n_2 \left(2\beta\sqrt{s_{12}} - f_1\right)}{\sqrt{s_{22}}\left(2\beta\sqrt{s_{12}} - f_1 + 2\beta\sqrt{s_{22}} - f_2\right)^2} + \frac{\delta n_{21}\sqrt{s_{12}}}{2\left(\sqrt{s_{12}} + \sqrt{s_{22}}\right)^2\sqrt{s_{22}}} - 1 - \lambda_2 = 0 \quad (33)$$

3)

$$\lambda_2 \geq 0,\ s_{21} + s_{22} = \xi_2 \quad \text{or} \quad \lambda_2 = 0,\ s_{21} + s_{22} \leq \xi_2 \quad (34)$$

4)

$$f_2 \geq 0,\ s_{21} \geq 0,\ s_{22} \geq 0.$$

C. Solving for Equilibrium

To solve the system and find its equilibrium values, we need to solve the eight simultaneous equations (27)-(34) for the eight variables s_{11}, s_{12}, s_{21}, s_{22}, f_1, f_2, λ_1 and λ_2. Here, λ_1 and λ_2 are the Lagrangian multipliers for Provider 1 and Provider 2, respectively. Once we solve this system of eight equations, we will obtain the closed form solutions for the providers' strategies as a function of ξ_1, ξ_2 and δ. Knowing these strategies, the government can then solve its optimization problem and find the optimal ξ_1 and ξ_2.