Pay or Perish: The Economics of Premium Peering

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***Abstract*— As the Internet continues to evolve, traditional peering agreements cannot accommodate the changing market conditions. Premium peering has emerged where access providers (APs) charge content providers (CPs) for premium services beyond best-effort connectivity. Although prioritized peering raises concerns about net neutrality, the U.S. FCC exempted peering agreements from its recent ruling, as it falls short of background in the Internet peering context. In this paper, we consider the premium peering options provided by APs and study whether CPs will choose to peer. Based on a novel choice model of complementary services, we characterize the market shares and utilities of the providers under various peering decisions and identify the value of premium peering for the CPs that fundamentally determine CPs' peering decisions. We find that high-value CPs have *peer pressure* when low-value CPs peer; however, low-value CPs behave oppositely. The peering decisions of the high-value and low-value CPs are substantially influenced by their baseline market shares and user stickiness, respectively, but not vice versa.**

***Index Terms*— Internet economics, premium peering, choice model, complementary services.**

I. Introduction

THE Internet consists of thousands of interconnected commercial networks, which make autonomous peering decisions with one another so as to make individual profits. Early business agreements between connecting networks were primarily in the form of either a customer-supplier or settlement-free peering relationship [13]. In the former form, suppliers, often higher tier Internet service providers (ISPs), sell *transit* services to lower tier networks such as content and access providers; in the latter form, networks with similar sizes and traffic characteristics such as the Tier-1 ISPs exchange traffic without billing each other. However, since a decade ago, these simple forms of peering no longer accommodate to the changing market conditions and industrial organization of the Internet. On October 5th 2005, Level 3 unilaterally terminated its settlement-free peering with Cogent [6], resulting in at least 15% of the Internet being unreachable for the customer networks which utilized either for Internet access. This de-peering happened because Cogent generated more outbound than inbound traffic on its peering connection with Level 3, breaking the traffic ratio requirement of settlement-free peering; and therefore, Level 3 wanted compensations under a new agreement. The situation is further complicated by the emergence of video streaming giant Netflix, which now accounts for up to 34% of peak U.S. downstream traffic [14]. As most Netflix customers used Comcast, the largest U.S. broadband provider, as their last-mile access provider, after Level 3 obtained a contract to deliver Netflix's traffic, Comcast started to see a surge in the inbound traffic from Level 3. On November 19, 2010, Comcast informed Level 3 that it will demand a recurring fee from Level 3 to deploy necessary capacities for streaming online movies to Comcast's customers [34]. Despite its reluctance, Level 3 has acquiesced in the new fees, which totally *reversed* the nominal customer-supplier relationship where the Tier-1 Level 3 should have received payment for connectivity as a supplier.

This emerging form of premium peering (also called paid peering [13]) is mainly driven by the new requirements from delay sensitive content providers that value service quality more than mere best-effort connectivity, which can often be obtained by publicly peering at Internet exchange points. As a type of paid prioritization, premium peering raises concerns about net neutrality [48]; however, the FCC's recent Open Internet Order [2] specifically exempts existing peering arrangements, because the FCC feels that it lacks in-depth background "in the Internet traffic exchange context." *In this paper, we analyze the economics of premium peering so as to understand whether or not networks will engage in it and the driving forces behind these decisions.* In particular, we consider the two-sided market structure [4], [40] of the Internet, where Internet access providers (APs) provide a platform connecting end-users to content providers (CPs) and study CPs' peering decisions when premium peering options are provided by the APs. We focus on such a CP-AP model for a couple of reasons. First, the Internet topology has been *flattening* [10] as large CPs, e.g., Google and Microsoft, are deploying wide-area infrastructures in order to bring content closer to users and bypass Tier-1 ISPs on many paths [15]. Recently, Netflix even agreed to pay APs such as Comcast [38] and Verizon [16] directly for smoother streaming. Second, most disputes over premium peering, e.g., Cogent-Telia, Cogent-Sprint [32], Netflix-Comcast and Netflix-Verizon [25], were centered around an AP charging a counterparty that specializes in content distribution. We use the term CP in a broad sense that it includes content delivery networks (CDNs), e.g., Akamai [37], transit ISPs, e.g., Level 3, and content hosting ISPs, e.g., Cogent. We model users' choices of providers under different peering structures among CPs and APs, and analyze the impact of various metrics, e.g., the baseline market share and user stickiness of the providers and APs' premium peering prices, on CPs' peering decisions. Our contributions and conclusions are as follows.

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- We build a novel choice model that takes AP and CP as complementary services (Section III) and characterize their market shares (Theorem 1) and utilities (Theorem 3 and 7) under various peering and market structures.
- We identify an intrinsic *value of premium peering* (VoPP) (Equation 9) and its generalization (Theorem 2 and 8) that fundamentally determine CPs' peering decisions.
- We find that high-value CPs have *peer pressure* when low-value CPs peer; however, low-value CPs behave oppositely. The peering decisions of the low- and high-value CPs are substantially influenced by their user stickiness and baseline market shares, respectively, but not vice versa (Section V and explained by Theorem 5).
- We find that whether an AP is peered depends on how sticky its users are and how cheap its competitors are (Theorem 6 and Section V), where the baseline market share plays a role of a scaling factor (Theorem 6) and determines the effective price if CPs peer (Theorem 5).
- We show that Nash equilibrium may not be unique or exist even under a two-CP case, depending on the CPs' market shares and relative user stickiness (Theorem 9).

Our results help understand how existing peering happened and predict how future peering outcomes will emerge. The new knowledge can help providers make informed premium peering decisions and guide regulators to design better policies to address net neutrality issues in the interconnection context.

II. Related Work

Huston [19] studied early interconnection settlements and concluded that customer-supplier and settlement-free were the only stable peering models for the Internet at the time. Faratin *et al.* [13] showed the emergence of paid peering and partial transit, as a result of the heterogeneity of networks. Based on coalition game theory, Ma *et al.* [27], [28] predicted that a reverse customer-supplier would emerge, since the existing settlements are not be stable. D'Ignazio and Giovannetti [12] empirically showed discriminations in the Internet peering using dataset from London Internet Exchange Point (LINX). Ma [26] and Ma *et al.* [29] studied the usage-based pricing of the access providers and the general evolution of the Internet economic ecosystem, respectively. Ma and Misra [30], Tang and Ma [46], and Ma *et al.* [31] studied the impact of paid prioritization from the net neutrality perspective. We focus on premium peering provided by APs, which creates the reverse customer-supplier relationship between APs and higher-tier networks under a form of discrimination.

Individual peering agreements are driven by economics and influence the topology of the Internet. Chang *et al.* [9] modeled the decision processes of peering and studied the evolution of the AS-level topology. Dhamdhere and Dovrolis [10] showed that the Internet topology has transitioned from a transit hierarchy to a peering mesh. Lodhi *et al.* [21] proposed an agent-based model to simulate the network formation process. These work used unstructured graph models and relied on quantitative simulations for evaluation. Many work [5], [20], [22], [23], [45], including ours, used structured models to study the peering relationship between networks. Lodhi *et al.* [22], [23] used a three-tier model that includes a top-tier transit ISP, Tier-2 ISPs and APs, and studied the peering decision of the transit ISP [22] and Tier-2 ISPs [23]. Due to the model complexity, they relied on agent-based simulation for evaluation. Tan *et al.* [45] studied the peering between two backbone ISPs; while, Badasyan and Chakrabarti [5] focused on the peering between two Tier-2 ISPs, both of which are connected by a Tier-1 provider. Jahn and Prüfer [20] studied a more general peering between ISPs, both of which can be either Tier-1 or Tier-2. These work focused on a particular pair of networks and derived analytical results. We focus on a two-sided structure of complementary service markets with competitions on both sides and derive analytical results for premium peering between CPs and APs.

Dhamdhere *et al.* [11] proposed a value-based quantitative framework to study peering agreements, and determined the globally fair, optimal and stable peering prices between two networks. Recently, Gyarmati *et al.* [17] used a churn model to study premium peering and determined the fair prices based on the Nash Bargaining solution [35]. Similar to [11], we propose a value, i.e., VoPP, for premium peering; similar to the churn model [17], we use a novel choice model to capture the user behaviors and market shares. However, our analysis focuses on the non-cooperative scenarios where APs set prices and CPs determine whether or not to peer.

Finally, similar to many discrete choice models, e.g., the multinomial logit model [33], we build our choice model based on the Luce's choice axiom [24], which has deep connections with other models, e.g., independent race models [8] and the classic theory of demand based on revealed preference [18]. Instead of focusing on a single market, we build our choice model upon a two-dimensional complementary service [43] market, under which users need to choose a pair of complementary service providers, i.e., a CP and an AP.

III. Model

We consider an Internet content market such as on-demand video streaming in a geographical region. To obtain content, any user needs a CP and an AP. In this section, we model these CPs and APs as two complementary services for the users such that each user chooses a pair of CP and AP. Based on a novel choice model of the users, we characterize the market shares of the providers, through which we further determine their utilities under various peering decisions.

A. Complementary Choices Model $(\mathcal{N}, M)$

We define $\mathcal{N} \triangleq \{1, \cdots, N\}$ and $\mathcal{M} \triangleq \{1, \cdots, M\}$ as the set of CPs and APs that serve the region. We assume that a premium peering option is provided by each AP and CPs decide whether or not to peer with each AP. We denote the peering relationship between CP $i \in \mathcal{N}$ and AP $j \in \mathcal{M}$ by $\theta_{ij} \in \{0, 1\}$, where $\theta_{ij} = 1$ indicates that a premium peering between i and j is established, otherwise $\theta_{ij} = 0$.[1] We define $\boldsymbol{\theta_i} \triangleq (\theta_{i1}, \cdots, \theta_{iM})$ and $\boldsymbol{\vartheta_j} \triangleq (\vartheta_{1j}, \cdots, \vartheta_{Nj})$

[1] Throughout the paper, we will simply say that a CP i peers or does not peer with an AP j if $\theta_{ij} = 1$ or $\theta_{ij} = 0$, respectively.

as the peering profile of CP i and AP j, respectively, and $\Theta \triangleq \{\theta_{ij} : i \in \mathcal{N}, j \in \mathcal{M}\}$ as the system's peering matrix.

We denote the *baseline* of CP i by $\phi_i \in (0, 1]$, which captures the *intrinsic* characteristics such as price and brand name, and models the market share of CP i when all CPs maintain the same *extrinsic* peering relationships with APs. For example, if all CPs maintain the same peering relationship with an AP j, i.e., $\boldsymbol{\vartheta}_j = \mathbf{0}$ or $\mathbf{1}$, the percentage ϕ_i of AP j's users will choose CP i from the set $\mathcal{N}$ of CPs. In probabilistic choice models [33], ϕ_i can also be interpreted as the probability that any user will choose CP i among the alternatives that maintain the same peering condition. Likewise, we denote the baseline of AP j by $\psi_j \in (0, 1]$, i.e., the market share of AP j when all APs maintain the same peering relationships with CPs. We define column vectors $\boldsymbol{\phi} \triangleq (\phi_1, \cdots, \phi_N)^T$ and $\boldsymbol{\psi} \triangleq (\psi_1, \cdots, \psi_M)^T$ as the baseline market share distributions with $\sum_{i\in\mathcal{N}} \phi_i = \sum_{j\in\mathcal{M}} \psi_j = 1$.

In practice, users may choose services from constrained sets of CPs and APs. It might be because certain providers are not available to the users or cannot satisfy their requirements. In general, we denote a set of choice pairs by $\mathcal{L}$. Based on the baseline market shares of the providers, we make the following assumption on the users' choices.

Assumption 1: Given a nonempty set $\mathcal{L}$ of available choices, a user chooses a choice pair $l = (i, j) \in \mathcal{L}$ with probability

$$\mathbb{P}_{\mathcal{L}}\{l = (i, j)\} = \frac{\phi_i \psi_j}{\sum_{(n,m)\in\mathcal{L}} \phi_n \psi_m}. \qquad (1)$$

Under Assumption 1, if $\mathcal{L}$ equals the choice set $\mathcal{N} \times M$, the probability of choosing (i, j) equals $\phi_i\psi_j$, which is consistent with our notion of *baseline*. Furthermore, by Luce's choice axiom [24], the proportional form in (1) is also necessary for guaranteeing an *independence from irrelevant alternatives* (IIA) property: the probability of selecting one item over another from a pool of many items is not affected by the presence or absence of other items in the pool.

If CP i stops the premium peering relationship with AP j such that service quality cannot be guaranteed, their users might switch to alternative providers in the market. We assume that given a set $\mathcal{O}$ of available options to choose from, each user will switch to a better pair of providers under premium peering in the set $\mathcal{L}(\Theta|\mathcal{O}) \triangleq \{(k, l) \in \mathcal{O} : \theta_{kl} = 1\}$ based on Assumption 1, but will stay with the current choice (i, j) if no premium peering exists, i.e., $\mathcal{L}(\Theta|\mathcal{O}) = \emptyset$.

Besides the baselines ϕ_i and ψ_j, we denote the stickiness of the users of CP i and AP j by $\alpha_i \in [0, 1]$ and $\beta_j \in [0, 1]$. Based on the stickiness of the providers, the set $\mathcal{O}$ of available options to any user of (i, j) is assumed as follows.

Assumption 2: The set $\mathcal{O}$ of available options of any user of the choice pair (i, j) satisfies

$$\mathcal{O} = \begin{cases} \{(i, j)\} & \text{with probability } \alpha_i\beta_j, \\ \{i\} \times \mathcal{M} & \text{with probability } \alpha_i(1 - \beta_j), \\ \mathcal{N} \times \{j\} & \text{with probability } (1 - \alpha_i)\beta_j, \\ \mathcal{N} \times \mathcal{M} & \text{with probability } (1 - \alpha_i)(1 - \beta_j). \end{cases}$$

Assumption 2 interprets the stickiness α_i and β_j as the percentage of users that will stay with CP i and AP j, respectively, when the peering relationship between CP i and AP j deteriorates, i.e., θ_{ij} changes from 1 to 0. In economics terms, the stickiness models the *elasticity of user demand* influenced by the provider's peering decisions.

As before, we define the vectors $\boldsymbol{\alpha} \triangleq (\alpha_1, \cdots, \alpha_N)$ and $\boldsymbol{\beta} \triangleq (\beta_1, \cdots, \beta_M)$ as the user stickiness of the CPs and APs, respectively. We also define $\bar{\alpha}_i \triangleq 1-\alpha_i$ and $\bar{\beta}_j \triangleq 1-\beta_j$ as the elastic proportion of users of the providers, $\bar{\theta}_{ij} \triangleq 1 - \theta_{ij}$ as the reverse peering relationship, and denote the corresponding vectors and matrix by $\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{\beta}}, \bar{\boldsymbol{\theta}}_i, \bar{\boldsymbol{\vartheta}}_j$ and $\bar{\Theta}$. In summary, our complementary choices model $(\mathcal{N}, M)$ can be entirely specified by a quadruple of vectors $(\boldsymbol{\phi}, \boldsymbol{\psi}, \boldsymbol{\alpha}, \boldsymbol{\beta})$. We denote the total market size by X. Based on the assumptions of user choice and stickiness (Assumption 1 and 2), we characterize the number of users of (i, j), denoted by X_{ij}, as a function of the peering matrix Θ of the system as follows.

Theorem 1: For a system $(\mathcal{N}, M)$ with a peering matrix Θ, the number of users of any pair (i, j) of CP and AP can be expressed as $X_{ij}(\Theta) = \rho_{ij}(\Theta)\phi_i\psi_j X$, where 1) if $\theta_{ij} = 0$,

$$\rho_{ij}(\Theta) = \alpha_i\beta_j + \alpha_i\bar{\beta}_j \mathbf{1}_{\{\boldsymbol{\theta}_i=\mathbf{0}\}} + \bar{\alpha}_i\beta_j \mathbf{1}_{\{\boldsymbol{\vartheta}_j=\mathbf{0}\}} + \bar{\alpha}_i\bar{\beta}_j \mathbf{1}_{\{\Theta=\mathbf{0}\}},$$

where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function, and 2) if $\theta_{ij} = 1$,

$$\rho_{ij}(\Theta) = 1 + \frac{\alpha_i \bar{\boldsymbol{\theta}}_i \boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\theta}_i \boldsymbol{\psi}} + \frac{\beta_j \bar{\boldsymbol{\vartheta}}_j \boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}}{\boldsymbol{\vartheta}_j \boldsymbol{\phi}} + \frac{\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}{}^T \bar{\Theta} \boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\phi}^T \Theta \boldsymbol{\psi}},$$

where we define the vectors $\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}} = \bar{\boldsymbol{\alpha}} \circ \boldsymbol{\phi}$ and $\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}} = \bar{\boldsymbol{\beta}} \circ \boldsymbol{\psi}$ as the Hadamard (element-wise) product of two vectors.

Theorem 1 derives the number of users X_{ij} of any pair (i, j) of complementary providers under the peering relationships Θ of the system. In particular, X_{ij} can be represented by its baseline market share $\phi_i\varphi_j X$ of users multiplied by a weighting factor $\rho_{ij}(\Theta)$, which is a function of the peering matrix Θ and the providers' user stickiness α_i and β_j. If CP i does not use premium peering with AP j, i.e., $\theta_{ij} = 0$, the pair (i, j) of providers can keep the proportion $\alpha_i\beta_i$ of their sticky users and possibly some share of their non-sticky users, e.g., the proportion of users $\alpha_i\bar{\beta}_j$ that are sticky to CP i but non-sticky to AP j if CP i does not use premium peering with any AP, i.e., $\boldsymbol{\theta}_i = \mathbf{0}$. If CP i uses premium peering with AP j, i.e., $\theta_{ij} = 1$, they will keep all their original users and obtain a baseline-weighted share of the non-sticky users of the providers that do not use premium peering. Based on Theorem 1, the market share of any CP i and AP j can be calculated as $\sum_{j\in\mathcal{M}} X_{ij}$ and $\sum_{i\in\mathcal{N}} X_{ij}$, respectively. As a sanity check, one can easily verify that when $\Theta = \mathbf{0}$ or $\mathbf{1}$, any coefficient ρ_{ij} equals 1, and the baselines ϕ_i and ψ_j truly represent the market shares of the providers.

Corollary 1 (Monotonicity): Given any Θ and for any pair (i, j), $X_{ij}(\widetilde{\Theta}) \le X_{ij}(\Theta)$ for all $\widetilde{\Theta} \ge \Theta$[2] with $\tilde{\theta}_{ij} = \theta_{ij}$.

Corollary 1 intuitively shows that when the peering relationship between CP i and AP j does not change, the users of (i, j) will be non-increase if more premium peering relationships emerge as alternatives to users in the market.

Corollary 2 (Additivity): For any $n \notin \mathcal{N} \supseteq \mathcal{N}_0$ and $m \notin \mathcal{M} \supseteq \mathcal{M}_0$, let $\widetilde{\mathcal{N}} \triangleq \mathcal{N}\backslash\mathcal{N}_0 \cup \{n\}$ and $\widetilde{\mathcal{M}} \triangleq \mathcal{M}\backslash\mathcal{M}_0 \cup \{m\}$ denote

[2]When we use binary operators like $\le$ or $\ge$ for vectors or matrices, we mean that the operator holds component-wise for the entities.

the new sets of providers where the subsets $\mathcal{N}_0$ and $\mathcal{M}_0$ are replaced by the providers n and m, respectively. Let $\widetilde{X}_{ij}$ denote the number of users of (i, j) under $(\widetilde{\mathcal{N}}, \widetilde{\mathcal{M}})$. If $(\boldsymbol{\theta_i}, \alpha_i) = (\boldsymbol{\theta_n}, \alpha_n)$, $\forall i \in \mathcal{N}_0$, $(\boldsymbol{\vartheta_j}, \beta_j) = (\boldsymbol{\vartheta_m}, \beta_m)$, $\forall j \in \mathcal{M}_0$, and $(\phi_n, \psi_m) = \left(\sum_{i\in\mathcal{N}_0} \phi_i, \sum_{j\in\mathcal{M}_0} \psi_j\right)$, then

$$\widetilde{X}_{nm} = \sum_{i\in\mathcal{N}_0}\sum_{j\in\mathcal{M}_0} X_{ij}; \quad \widetilde{X}_{ij} = X_{ij},\ \forall i \neq n, j \neq m;$$
$$\widetilde{X}_{nj} = \sum_{i\in\mathcal{N}_0} X_{ij},\ \forall j \neq m; \text{ and } \widetilde{X}_{im} = \sum_{j\in\mathcal{M}_0} X_{ij},\ \forall i \neq n.$$

Corollary 2 states that if there exists multiple CPs (or APs) that have the same stickiness and use the same peering profile, then they could be conceptually merged as a single CP (or AP) without affecting the market shares of other providers.

B. Premium Peering and Utility Model

Pricing takes various forms for the Internet in practice. Wireline access providers often charge flat-rates [3], while tiered schemes have recently been adopted by major U.S. broadband providers such as Verizon [41] and AT&T [47]. Internet transit services, however, are usually charged based on peak rates, e.g., the 95th percentile measurement [44]. Although peering pricing may also take various forms [36], we assume that each AP chooses its pricing structure and each CP determines whether or not to peer with each AP. Notice that although both the CP and AP sides are symmetric in the complementary choice model developed in the previous subsection, the premium peering agreements in the form of "take it or leave it" contracts are asymmetric. As CPs' premium peering decisions depend on the imposed prices of APs rather than their underlying costs, we will assume the prices of APs to be exogenous. Generally, we denote the per-user peering price of AP j by p_j. Under usage-based pricing, p_j models the charge for the throughput induced by an average user under premium peering; under flat-rate pricing, p_j captures each user's share of the total premium peering charges borne by the CPs. Similarly, we denote the per-user value of CP i generated under premium peering by q_i. We also assume that CP i has a lower per-user value $\delta_i q_i$ if premium peering is not used, where $\delta_i \in [0, 1]$ denotes the percentage of content value preserved under the inferior service quality without using premium peering. In practice, inelastic content, e.g., real-time multimedia, has much weaker preservability than elastic content, e.g., file download. Under any peering matrix Θ, we denote the utility of CP i by $U_i(\Theta)$ and the revenue of AP j by $R_j(\Theta)$, defined as

$$R_j(\Theta) \triangleq p_j \sum_{i\in\mathcal{N}} \theta_{ij} X_{ij}(\Theta) \quad \text{and} \quad U_i(\Theta) \triangleq \sum_{j\in\mathcal{M}} U_i^j(\Theta),$$

$$\text{where} \quad U_i^j(\Theta) \triangleq \begin{cases} q_i\delta_i X_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 0, \\ (q_i - p_j) X_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 1. \end{cases} \tag{2}$$

Each AP j's utility here is interpreted as the per-user revenue p_j multiplied by the total number of users under its premium peering relationship with all the CPs, i.e., $\sum_{i\in\mathcal{N}} \theta_{ij} X_{ij}(\Theta)$. Each CP i's utility is the sum over the utilities U_i^j generated with each AP j, which equals the number of users $X_{ij}(\Theta)$ multiplied by either its profit margin $q_i - p_j$ if premium peering is used or its discounted value $\delta_i q_i$ otherwise.

IV. Monopolistic Analysis

Although pure monopolistic markets are rare in practice, e.g., many users can choose between DSL and cable in the U.S., approximately 96% of the U.S. population has at most two wireline providers [1]. Furthermore, based on the FCC's definition of broadband service, i.e., at least 25Mbps downstream and 3Mbps upstream, more than 50 million Americans do not have access to high-speed wired Internet service and Comcast owns 56.8% of all U.S. broadband customers [7]. Meanwhile, Netflix traffic now accounts for 34% of North America's downloads during peak hours [14], which also shows that the content market is far from competitive.

We believe that many broadband and content markets are not competitive enough, and therefore, monopolistic cases are still relevant and worth exploring. From a methodical perspective, they represent special but important cases of general settings where competition may exist on both the CP-side and AP-side markets, and the monopolistic results will help understand more complicated cases such as complementary duopoly discussed in the next section. In particular, as a CP i's (an AP j's) baseline market share ϕ_i (φ_j) approaches 1, the complementary choice model converges to a monopolistic CP (AP) scenario smoothly. Furthermore, Corollary 2 also implies that if the user stickiness α_i (β_j) of the CPs (APs) are similar, they can be conceptually aggregated as a monopolistic CP (AP) if they employ the same peering strategy with the APs (CPs) on the other side of the market. When a CP or AP market is monopolistic, we omit the subscript i or j and use general notation p, q, δ, $\boldsymbol{\theta}$, $\boldsymbol{\vartheta}$, U and R to denote the peering price, content value, preservability, peering profiles and utility functions of the monopoly providers, respectively. Notice that for a monopoly provider, its baseline, i.e., θ or ψ, and stickiness, i.e., α or β, equal 1 by definition.

A. Complementary Monopoly

We start with the simplest case of one CP and one AP, i.e., $M = N = 1$. As both the AP and CP markets are monopolistic, the market structure is *complementary monopoly* [43]. Under this model, the matrix Θ becomes a single peering decision: $\Theta = 0$ or 1. Whether the CP will use premium peering depends on how much the AP charges for it. In the following theorem, we derive the noncooperative pricing and peering strategies for the providers, as well as the cooperative solutions based on the Nash bargaining solution [35] and the Shapley value [42], so as to understand what the *fair* peering price should be in principle for the AP to charge.

Theorem 2: Under any peering price p and peering decision Θ, the CP's utility $U(\Theta)$ and the AP's revenue $R(\Theta)$ satisfy

$$\begin{cases} U(\Theta) = \delta q X \text{ and } R(\Theta) = 0 & \text{if} \quad \Theta = 0, \\ U(\Theta) = (q - p)X \text{ and } R(\Theta) = pX & \text{if} \quad \Theta = 1. \end{cases} \tag{3}$$

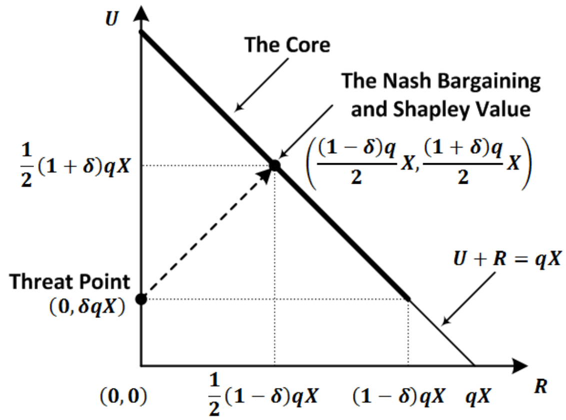


Fig. 1. Nash bargaining and Shapley value under fair price $p = \frac{1}{2}(1-\delta)q$.

To maximize its utility, the CP will choose to use premium peering if and only if the AP's price satisfies $p \le (1-\delta)q$. Both the Shapley value and the Nash bargaining solution are achieved at the price $p = \frac{1}{2}(1-\delta)q$, resulting in the utilities

$$R = \frac{1}{2}(1-\delta)qX \quad \text{and} \quad U = \frac{1}{2}(1+\delta)qX.$$

Theorem 2 states that the CP will use premium peering if the price is lower than $(1-\delta)q$, because its gain from premium peering equals $qX - \delta qX = (1-\delta)qX$. Both the Nash bargaining and the Shapley value lead to the same price $p = \frac{1}{2}(1-\delta)q$, under which both the CP and AP evenly share this extra gain $(1-\delta)qX$ from premium peering.

Figure 1 visualizes the solutions in a two-dimensional utility space, where the x-axis and y-axis show R and U, respectively. When premium peering is not used, the utilities correspond to the *Threat Point* $(R, U) = (0, \delta qX)$; when premium peering is established, the achievable utilities lie on the line $U+R = qX$ with AP's revenue $R = pX$. The Nash bargaining solution is achieved by moving the *Threat Point* along the 45-degree line to reach a solution on the line $U + R = qX$. The thick line segment shows the *core*, i.e., the set of stable solutions where no party will deviate, which can be achieved by the set of prices $p \in [0, (1-\delta)q]$. The Shapley value is located at the center of the core and coincides with the Nash bargaining solution. Notice that the model assumed zero cost of premium peering for the AP, and after taking these costs into consideration, the fair pricing will increase and the sharing solution will move downward.

Because the value threshold $(1-\delta)q$ determines whether or not premium peering is worth using from the CP's perspective, we define the per-user *intrinsic value of premium peering* (VoPP) of the monopoly CP as $\upsilon \triangleq (1-\delta)q$.

B. Monopolistic Content Providers

We continue with the scenarios of a monopoly CP, i.e., $N = 1$. Under fixed APs' prices $\boldsymbol{p} \triangleq \{p_1, \cdots, p_M\}$, the CP's utility $U(\Theta)$ depends on its peering strategy $\boldsymbol{\theta} = \Theta$. Under any strategy $\boldsymbol{\theta}$, we denote the percentage of sticky users of the APs whose premium peering services are not used by the monopoly CP by $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})$, defined as

$$\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta}) \triangleq \begin{cases} 1 & \text{if } \boldsymbol{\theta} = \boldsymbol{0}, \\ (\bar{\boldsymbol{\theta}} \circ \boldsymbol{\beta})\boldsymbol{\psi} = \sum_{j\in\mathcal{M}} \bar{\theta}_j \beta_j \psi_j & \text{if } \boldsymbol{\theta} \neq \boldsymbol{0}. \end{cases} \tag{4}$$

We denote the $\boldsymbol{\psi}$-weighted average price of the APs that the CP peers with under strategy $\boldsymbol{\theta}$ by $\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta})$, defined as

$$\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta}) \triangleq \begin{cases} 0 & \text{if } \boldsymbol{\theta} = \boldsymbol{0}, \\ \dfrac{(\boldsymbol{p} \circ \boldsymbol{\theta})\boldsymbol{\psi}}{\boldsymbol{\theta}\boldsymbol{\psi}} = \dfrac{\sum_{j\in\mathcal{M}} \theta_j p_j \psi_j}{\sum_{j\in\mathcal{M}} \theta_j \psi_j} & \text{if } \boldsymbol{\theta} \neq \boldsymbol{0}. \end{cases} \tag{5}$$

Notice that when $\boldsymbol{\theta} = \boldsymbol{0}$, the CP does not use premium peering with any AP and therefore, all users stick with their original APs, i.e., $\Psi_{\boldsymbol{\beta}}(\boldsymbol{0}) = 1$. Since no AP charges the CP, we define the average price $\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{0}) = 0$ for convenience.

Theorem 3: Under any strategy $\boldsymbol{\theta}$, AP j's revenue satisfies

$$R_j(\boldsymbol{\theta}) = \begin{cases} p_j[1 - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]\dfrac{\psi_j}{\boldsymbol{\theta}\boldsymbol{\psi}}X & \text{if } \theta_j = 1, \\ 0 & \text{if } \theta_j = 0. \end{cases} \tag{6}$$

The CP's utility satisfies $U(\boldsymbol{\theta}) = \left[q - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon\right] X - R(\boldsymbol{\theta})$, where the CP's aggregate payment to all the APs satisfies

$$R(\boldsymbol{\theta}) \triangleq \sum_{i\in\mathcal{M}} R_j(\boldsymbol{\theta}) = \left[1 - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\right] \bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta})X.$$

Theorem 3 expresses the CP's utility and payment to the APs as functions of the peering strategy $\boldsymbol{\theta}$. In particular, $R(\boldsymbol{\theta})$ equals the number of non-sticky users of the CP, i.e., $[1 - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]X$, multiplied by the per-user price $\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta})$, i.e., the $\boldsymbol{\psi}$-weighted average price of the peered APs. This shows that an AP j's baseline ψ_j plays a major role in determining the effective peering price for the CP, if it chooses to peer with AP j. Besides payment, the CP's revenue is reduced from its maximum qX by $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon X$, which can be interpreted as the number of sticky users multiplied by the per-user intrinsic VoPP υ that is lost without premium peering.

Corollary 3: Let $U^*(\boldsymbol{p})$ denote the CP's maximum achievable utility under APs' prices $\boldsymbol{p}$. $U^*(\boldsymbol{p})$ is continuous, non-increasing and convex in $\boldsymbol{p}$. In particular, if $p_j = \upsilon$ for all $j \in \mathcal{M}$, then $U^*(\boldsymbol{p}) = U(\boldsymbol{\theta}) = \delta qX$ for all strategies $\boldsymbol{\theta}$.

Corollary 3 states that the CP's maximum utility is non-increasing in the prices $\boldsymbol{p}$ and decreases sub-linearly as prices increase. Intuitively, when the CP chooses not to peer with an AP j, it does not lose the intrinsic VoPP υ for all $\psi_j X$ users, but still obtain a full value q from $\bar{\beta}_j \psi_j X$ non-sticky users that switch to alternative APs with which the CP peers. It also shows that if all APs' prices equal the intrinsic VoPP υ, the CP will be indifferent among all peering strategies.

Next, we study the CP's optimal peering strategy and show that the intrinsic VoPP υ is a crucial threshold in determining whether or not premium peering will be used.

Theorem 4: Under any fixed prices $\boldsymbol{p}$, let $\boldsymbol{\theta}^*$ be an optimal strategy, i.e., $U(\boldsymbol{\theta}^*) = U^*(\boldsymbol{p})$. If $\boldsymbol{p} \ge \boldsymbol{\upsilon}$, $U(\boldsymbol{\theta}^*) = U(\boldsymbol{0})$; otherwise, $U(\boldsymbol{\theta}^*) > U(\boldsymbol{0})$ and $\theta_j^* = 0$ for all $p_j \ge \upsilon$.

Theorem 4 classifies the prices into two cases: a trivial case of $\boldsymbol{p} \ge \boldsymbol{\upsilon}$ where $\boldsymbol{\theta}^* = \boldsymbol{0}$ maximizes the CP's utility and a non-trivial case where there exists an AP with price lower than υ,

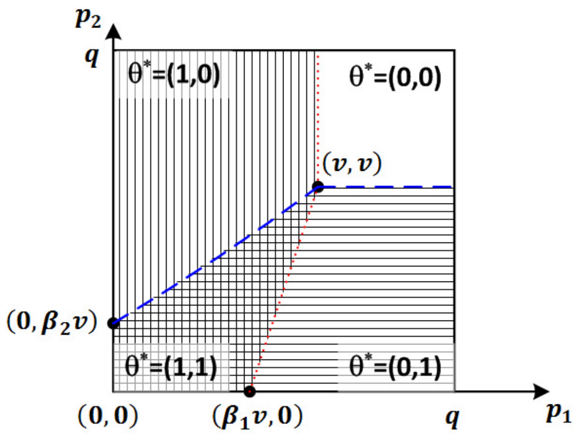


Fig. 2. Optimal peering strategy $\boldsymbol{\theta}^*$ under two APs.

under which an optimal strategy $\boldsymbol{\theta}^* \neq \mathbf{0}$ achieves higher utility than $U(\mathbf{0})$. Intuitively, APs with prices higher than v will not be used under an optimal strategy, which implies that the $\boldsymbol{\psi}$-weighted average price $(\boldsymbol{p} \circ \boldsymbol{\theta}^*)\boldsymbol{\psi}/\boldsymbol{\theta}^*\boldsymbol{\psi}$ must be no greater than v. However, $p_j < v$ does not guarantee that AP j will be used under an optimal strategy.

Theorem 5: Let $\tilde{\boldsymbol{\theta}}$ be a strategy with $\tilde{\theta}_j = 0$ for some AP j and $\boldsymbol{\theta}$ equal $\tilde{\boldsymbol{\theta}}$ except $\theta_j = 1$. $U(\boldsymbol{\theta}) \geq U(\tilde{\boldsymbol{\theta}})$ if any only if

$$\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta}) \leq \frac{1 - \Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}})}{1 - \Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}}) + \beta_j \psi_j} \bar{p}_{\boldsymbol{\psi}}(\tilde{\boldsymbol{\theta}}) + \frac{\beta_j \psi_j}{1 - \Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}}) + \beta_j \psi_j} v.$$

Theorem 5 states that the CP can increase its utility by peering with an AP j if the induced effective price $\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta})$ is lower than a weighted sum of the existing price $\bar{p}_{\boldsymbol{\psi}}(\tilde{\boldsymbol{\theta}})$ and its intrinsic VoPP v. If $\tilde{\boldsymbol{\theta}} = \mathbf{0}$, the condition yields $p_j \leq v$; otherwise, it yields $[1 - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]\bar{p}_{\boldsymbol{\psi}}(\boldsymbol{\theta}) \leq [1 - \Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}})]\bar{p}_{\boldsymbol{\psi}}(\tilde{\boldsymbol{\theta}}) + \beta_j \psi_j v$, which requires the induced per-user payment to be no higher than the existing one plus the gain $\beta_j \psi_j v$ obtained from AP j's sticky users. This result implies that the peering value with any particular AP is no higher than the CP's intrinsic VoPP v under a competitive market of multiple APs.

Corollary 4: When $M = 2$, we denote the AP other than AP j by $\bar{j}$. The peering decision $\theta_j = 1$ is part of an optimal strategy if and only if $p_j \leq \beta_j v + \bar{\beta}_j \min\{p_{\bar{j}}, v\}$.

Corollary 4 fully characterizes the optimal peering strategy under two APs: if AP $\bar{j}$'s price is higher than v, AP j is used when $p_j \leq v$; otherwise, AP j is used when $p_j \leq \beta_j v + \bar{\beta}_j p_{\bar{j}} \leq v$. It shows that with two APs, whether an AP is used depends on 1) *intrinsically* how sticky its users are, and 2) *extrinsically* how cheap the alternative AP is.

Figure 2 visualizes the optimal peering strategies under the price domain $[0, q]^2$. The condition in Corollary 4 can be written as $p_j \leq \min\{\beta_j v + \bar{\beta}_j p_{\bar{j}}, v\}$, the minimum of a linear function with slope $\bar{\beta}_j$ and the constant VoPP v, which are shown as the dashed blue and the dotted red boundaries. Both boundaries intersect at the utility-neutral point (v, v). When p_2 is below the dashed blue line, $\theta_2 = 1$ is optimal; when p_1 is left to the dotted red line, $\theta_1 = 1$ is optimal. Based on this figure, we can intuitively understand the role of the stickiness parameter β_j. For example, if $\beta_2 = 1$, the dashed blue line will becomes flat and $\theta_2 = 1$ will be optimal as long as $p_2 \leq v$, independent of p_1; if $\beta_2 = 0$, the first dashed blue line segment will start from the origin with slope 1 and $\theta_2 = 1$ will be optimal if $p_2 \leq \min\{p_1, v\}$. This explains that if an AP j totally controls its baseline market ψ_j, the situation is the same as complementary monopoly; if all its users are non-sticky, it will enter into a *price war* with competitor $\bar{j}$ to attract the CP for using premium peering. From the CP's perspective, APs with lower prices p_j and higher stickiness β_j are preferred for peering, while their baseline ψ_j plays a role of a scaling factor as shown below.

Theorem 6: If there exists two APs j and m that satisfy $p_j \psi_j > p_m \psi_m$ and $\beta_j \psi_j < \beta_m \psi_m$, then any peering strategy $\boldsymbol{\theta}$ satisfying $1 = \theta_j > \theta_m = 0$ is not optimal.

Theorem 6 intuitively states that under an optimal peering strategy, an inferior AP j will not be chosen if a superior AP m that has cheaper $\boldsymbol{\psi}$-weighted price $\psi_m p_m$ and higher sticky market share $\beta_m \psi_m$ is not chosen.

C. Monopolistic Access Providers

We now consider the scenarios of a monopoly AP, i.e., $M = 1$. Under the AP's price p, each CP i's utility $U_i(\Theta)$ depends on the peering profile $\Theta = \boldsymbol{\vartheta}$. Similar to the definition of $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})$ in Equation (4), under any strategy $\boldsymbol{\vartheta}$, we denote the percentage of sticky users of the CPs that do not peer with the monopoly AP by $\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})$, defined as

$$\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}) \triangleq \begin{cases} 1 & \text{if } \boldsymbol{\vartheta} = \mathbf{0}, \\ (\bar{\boldsymbol{\vartheta}} \circ \boldsymbol{\alpha})\boldsymbol{\phi} = \sum_{i \in \mathcal{N}} \bar{\vartheta}_i \alpha_i \phi_i & \text{if } \boldsymbol{\vartheta} \neq \mathbf{0}. \end{cases} \quad (7)$$

Theorem 7: Under any strategy $\boldsymbol{\vartheta}$, CP i's utility satisfies

$$U_i(\boldsymbol{\vartheta}) = \begin{cases} (q_i - p)[1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})] \dfrac{\phi_i}{\boldsymbol{\vartheta}\boldsymbol{\phi}} X & \text{if } \vartheta_i = 1, \\ q_i \delta_i [\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta} = \mathbf{0}\}}] \phi_i X & \text{if } \vartheta_i = 0. \end{cases} \quad (8)$$

The AP's revenue satisfies $R(\boldsymbol{\vartheta}) = [1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})]\, pX$.

Theorem 7 expresses the utility of the AP and CPs as functions of the peering strategy $\boldsymbol{\vartheta}$. Intuitively, $R(\boldsymbol{\vartheta})$ equals the number of non-sticky users of the AP, i.e., $[1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})]X$, multiplied by the per-user price p. Under premium peering, CP i's profit equals the per user margin $q_i - p$ multiplied by its share $\phi_i/\boldsymbol{\vartheta}\boldsymbol{\phi}$ of the number of non-sticky users of the AP; otherwise, it equals the discounted per-user value $\delta_i q_i$ multiplied by the number of its sticky users $[\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta} = \mathbf{0}\}}]\phi_i X$, where the factor $\bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta} = \mathbf{0}\}}$ reflects that even non-sticky users will stay with their CPs when no premium peering exists. This shows that a peering CP's baseline ϕ_i not only captures its intrinsic market share, but also plays a similar role in attracting the non-sticky users of the CPs that do not peer.

Given any peering strategy $\boldsymbol{\vartheta}$, we denote the strategies of all CPs except CP i by $\boldsymbol{\vartheta}_{-i}$. Theorem 7 shows that CP i's utility $U_i(\boldsymbol{\vartheta})$ depends not only on its own strategy ϑ_i, but all other CPs' peering decisions $\boldsymbol{\vartheta}_{-i}$ as well. We assume that given the price p, CPs make simultaneous peering decisions to maximize their utilities and define a Nash equilibrium as follows.

Definition 1 (Nash equilibrium): In a monopoly market of AP, a peering strategy profile $\boldsymbol{\vartheta}$ is a *Nash equilibrium* (NE) if and only if $U_i(\boldsymbol{\vartheta}) \geq U_i(\bar{\vartheta}_i; \boldsymbol{\vartheta}_{-i})$ for any CP $i \in \mathcal{N}$.

To generalize the intrinsic VoPP υ of a monopoly in a market of multiple CPs, we define the intrinsic VoPP of CP i as

$$\upsilon_i \triangleq (1 - \alpha_i \delta_i) q_i, \tag{9}$$

where $\alpha_i \delta_i q_i$ captures the value that can be kept without premium peering, as α_i portion of the users are sticky to CP i. Next, we characterize the Nash equilibrium based on CPs' *effective* VoPPs, a further generalization of CPs' intrinsic VoPPs under the market competition among them.

Theorem 8: For any $\boldsymbol{\vartheta}$, we define $\boldsymbol{\vartheta}_{i=1} \triangleq (\vartheta_i = 1; \boldsymbol{\vartheta}_{-i})$. A peering profile $\boldsymbol{\vartheta}$ is a Nash equilibrium if and only if

$$p \geq \tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i}), \ \forall \vartheta_i = 0 \quad \text{and} \quad p \leq \tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i}), \ \forall \vartheta_i = 1,$$

where $\tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i})$ denotes the effective VoPP, defined as

$$\tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i}) \triangleq \left[1 - \frac{\boldsymbol{\vartheta}_{i=1}\boldsymbol{\phi}}{1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1})} \left(\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}_{-i} = \mathbf{0}\}}\right) \delta_i \right] q_i.$$

In particular, $\tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i}) = \upsilon_i$ if $\boldsymbol{\vartheta}_{-i} = \mathbf{1}$ or $\boldsymbol{\alpha}_{-i} = \mathbf{1}$.

Theorem 8 characterizes the Nash equilibrium by specifying an effective VoPP $\tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i})$ for each CP i which determines whether or not CP i should peer with the monopoly AP under the strategies $\boldsymbol{\vartheta}_{-i}$ of all other CPs. The effective VoPP $\tilde{\upsilon}_i$ also generalizes the intrinsic VoPP υ_i in the sense that if $\boldsymbol{\vartheta}_{-i} = \mathbf{1}$ or $\boldsymbol{\alpha}_{-i} = \mathbf{1}$, all CPs except i will keep their baseline market shares and consequently, $\tilde{\upsilon}_i$ will coincide with CP i's intrinsic VoPP $\upsilon_i = (1 - \alpha_i \delta_i) q_i$. This is because when users are all sticky to their CPs, the peering decisions of the CPs become independent and the result is the same as that under the complementary monopoly case.

For $N = 2$, by defining $\hat{\upsilon}_i \triangleq \tilde{\upsilon}_i(\boldsymbol{\vartheta}_{-i} = 0)$, we can fully characterize the Nash equilibrium by Theorem 8 as

$$\begin{cases} \boldsymbol{\vartheta} = (1,1) \text{ is a Nash equilibrium iff } & p \leq \min\{\upsilon_1, \upsilon_2\}, \\ \boldsymbol{\vartheta} = (0,1) \text{ is a Nash equilibrium iff } & \upsilon_1 \leq p \leq \hat{\upsilon}_2, \\ \boldsymbol{\vartheta} = (1,0) \text{ is a Nash equilibrium iff } & \upsilon_2 \leq p \leq \hat{\upsilon}_1, \\ \boldsymbol{\vartheta} = (0,0) \text{ is a Nash equilibrium iff } & p \geq \max\left\{\hat{\upsilon}_1, \hat{\upsilon}_2\right\}. \end{cases}$$

If q_2 is sufficiently larger than q_1 such that the VoPPs satisfy $\min\{\upsilon_2, \hat{\upsilon}_2\} \geq \max\{\upsilon_1, \hat{\upsilon}_1\}$, the NE is determined as

$$\begin{cases} (1,1) \text{ is the unique NE if } & p < \upsilon_1, \\ (1,1) \text{ and } (1,0) \text{ are both NEs if } & p = \upsilon_1, \\ (1,0) \text{ is the unique NE if } & p \in (\upsilon_1, \hat{\upsilon}_2), \\ (1,0) \text{ and } (0,0) \text{ are both NEs if } & p = \hat{\upsilon}_2, \\ (0,0) \text{ is the unique NE if } & p > \hat{\upsilon}_2. \end{cases} \tag{10}$$

This shows that if the value q_i of the CPs are separate enough, under any price p, a unique equilibrium naturally emerges where the high-value CPs choose to use premium peering. However, if q_is are close, the existence and uniqueness of equilibrium cannot be guaranteed even for this two-CP case.

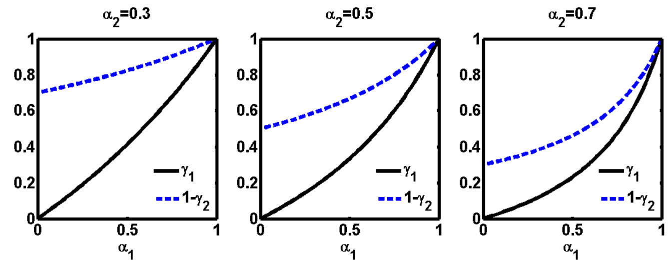


Fig. 3. Thresholds γ_1 and $1 - \gamma_2$ under various $\boldsymbol{\alpha}$.

Theorem 9: For $N = 2$, all the non-trivial cases of $\alpha_1 \alpha_2 < 1$ can be classified into three mutually exclusive categories:

$$(a) \begin{cases} \phi_1 > \gamma_1 \\ \phi_2 < \gamma_2 \end{cases}, \quad (b) \begin{cases} \phi_1 \geq \gamma_1 \\ \phi_2 \geq \gamma_2 \end{cases} \quad \textit{and} \quad (c) \begin{cases} \phi_1 < \gamma_1 \\ \phi_2 > \gamma_2 \end{cases},$$

$$\text{where} \quad \gamma_1 \triangleq \frac{\alpha_1 \bar{\alpha}_2}{1 - \alpha_1 \alpha_2} \quad \text{and} \quad \gamma_2 \triangleq \frac{\alpha_2 \bar{\alpha}_1}{1 - \alpha_1 \alpha_2}.$$

If we assume $\alpha_1 \delta_1 \geq \alpha_2 \delta_2$ without loss of generality and consider the case of $q_1 = q_2$, then we conclude that under (a), $\min\{\upsilon_2, \hat{\upsilon}_2\} \geq \max\{\upsilon_1, \hat{\upsilon}_1\}$; under (b), if $p \in \left(\max\{\hat{\upsilon}_1, \hat{\upsilon}_2\}, \upsilon_1\right)$, both $(0,0)$ and $(1,1)$ are Nash equilibria; and under (c), if $p \in \left(\max\{\upsilon_1, \hat{\upsilon}_2\}, \min\{\hat{\upsilon}_1, \upsilon_2\}\right)$ and $\sqrt{\bar{\alpha}_1}\phi_1 < \sqrt{\bar{\alpha}_2}\phi_2$, no Nash equilibrium exists.

Theorem 9 shows that there are two-CP scenarios under which zero or multiple Nash equilibria exist. In particular, three solution regions are classified by how large ϕ_i is compared to a threshold γ_i, which is a function of α_1 and α_2.

Figure 3 plots the thresholds γ_1 and $1 - \gamma_2$ against α_1 along the x-axis where α_2 is set to be 0.3, 0.5 and 0.7 in the three subfigures, respectively. Any distribution (ϕ_1, ϕ_2) can be represented by a point on the vertical segment [0, 1], and the curves γ_1 and $1 - \gamma_2$ visually partition the three solution regions: cases (a) and (c) correspond to the regions above $1 - \gamma_2$ and below γ_1, respectively, and case (b) corresponds to the region in between. We observe that when α_1 increases, γ_1 increases but γ_2 decreases. If we denote the CP other than CP i by $\bar{i}$, as the definitions of γ_1 and γ_2 are symmetric, in general, γ_i increases with α_i but decreases with $\alpha_{\bar{i}}$, and therefore, we can interpret γ_i as the *relative stickiness* of CP i. By assuming $q_1 = q_2$ and $\alpha_1 \delta_1 \geq \alpha_2 \delta_2$, CP 2 has a higher intrinsic VoPP, i.e., $\upsilon_2 \geq \upsilon_1$, and based on the interpretation of γ_i, we can understand the result of Theorem 9 as follows. Under case (a), CP 2 has a relatively small baseline but large user stickiness, and therefore, it always has more incentives to use premium peering to capture a non-negligible share of the non-sticky users of CP 1, and the Nash equilibrium follows Equation (10). Under case (c), on the contrary, CP 1 has more incentive to use premium peering if CP 2 does not; however, given the loss of market share under the strategy profile (1, 0), CP 2 will also use premium peering to regain its market share, under which CP 1 prefers not to use premium peering as it has a low intrinsic VoPP, and therefore, cannot capture the non-sticky users of CP 2. Thus, CP 1 (CP 2) will always be better off by using a peering strategy different from (same as) its competitor's, which leads to the non-existence of Nash

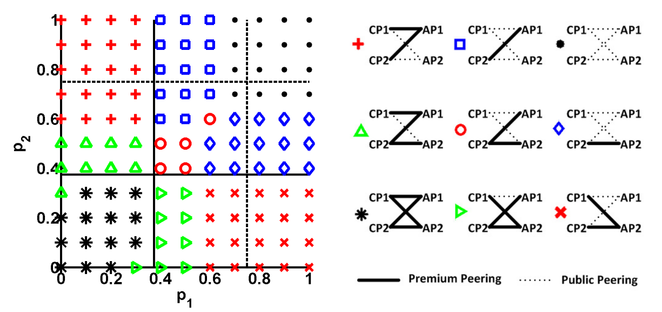


Fig. 4. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\phi} = \boldsymbol{\psi} = \boldsymbol{\delta} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.

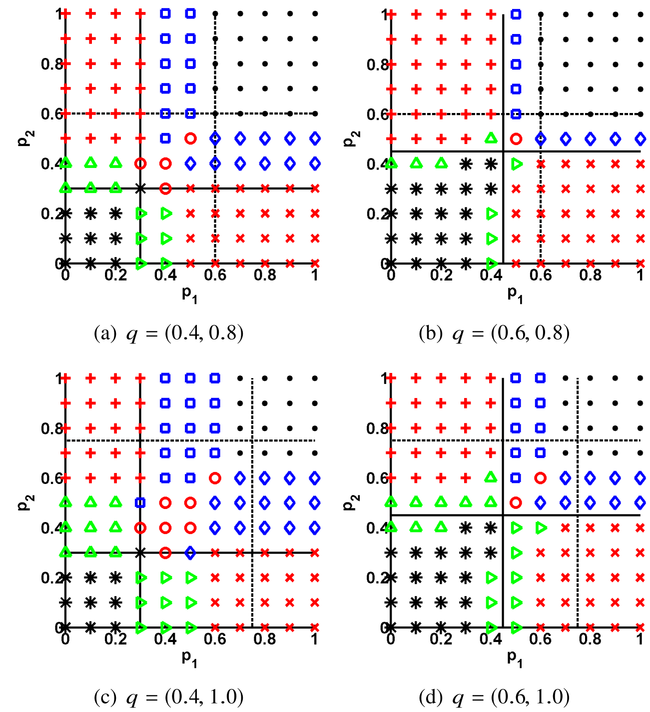
equilibrium. Under case (b), however, both the baseline and user stickiness of the CPs are close, and the best response of a CP is to mimic the strategy of the other CP and therefore, both $(0, 0)$ and $(1, 1)$ are Nash equilibria.

V. Complementary Duopoly

We further consider a complementary duopoly structure, i.e., $M = N = 2$, where two CPs and two APs compete in both sides of the market. As Corollary 2 shows that providers with similar user stickiness and peering strategies can be merged, the duopolistic model provides a first-order approximation of market competition from a provider's perspective such that all its competitors are considered as an aggregated provider that captures the remaining market share. As an AP j's baseline market share φ_j approaches 1, the complementary duopoly model converges to the monopolistic AP with $N = 2$ and Theorem 9 showed that neither the uniqueness nor the existence of Nash equilibrium can be guaranteed; and therefore, we conduct systematic numerical evaluations to understand this more complicated market structure. We start with a benchmark scenario where providers have the same baseline, i.e., $\boldsymbol{\phi} = \boldsymbol{\psi} = (0.5, 0.5)$, and the same user stickiness and content preservability, i.e., $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\delta} = (0.5, 0.5)$. Without loss of generality, we assume $q_1 \leq q_2$ and normalize $q_2 = 1$ for CP 2. We set $q_1 = 0.5$ for CP 1.

Figure 4 visualizes the Nash equilibria[3] when APs' prices p_1 and p_2 vary along the x- and y-axis, respectively. As $q_2 > q_1$ and the values are separate enough, 9 of the 16 possible peering profiles, shown in the legends in the right subfigure, become the unique Nash equilibrium under various prices in our numerical evaluation. Intuitively, when prices are low (high), any (neither) CP peers with any AP; when prices are in a mid-range, i.e., 0.4 to 0.6, CP 2 peers with both APs while CP 1 does not peer with any due to its low value q_1. Under any fixed price $p_{\bar{j}}$, we observe that as p_j increases, CP 1 first de-peers with AP j, followed by CP 2. These two boundary prices under which CPs start to de-peer correspond to the CPs' effective VoPPs with AP j under the CPs' peering decisions $\boldsymbol{\vartheta}_{\bar{j}}$ with the competitor AP $\bar{j}$ and its price $p_{\bar{j}}$. To compare these effective VoPPs of a CP with its intrinsic VoPP, we plot $p_1 = \upsilon_1$ and $p_1 = \upsilon_2$ ($p_2 = \upsilon_1$ and $p_2 = \upsilon_2$) in a solid and a dashed vertical (horizontal) line, respectively. We observe that any CP's effective VoPP is smaller by its intrinsic VoPP under competition. Furthermore, any CP's effective VoPP with an AP j will move towards a lower value if the other AP's price $p_{\bar{j}}$ decreases. For example, given $p_j = 0.3$ (0.6), CP 1 (2) peers with AP j only if $p_{\bar{j}} > 0$ (0.5). This is because when there exists a cheap alternative AP $\bar{j}$, even CPs do not peer with AP j, they can still attract the elastic users of AP j by peering with AP $\bar{j}$ under low costs; and therefore, the effective VoPP with AP j will decrease. We also observe that the effective VoPP of the low-value CP 1 is very close to its intrinsic VoPP υ_1, while that of CP 2 is not close to υ_2. Because CP 2 always peers with APs at the price region of CP 1's effective VoPP, similar to the monopolistic AP scenario of $\tilde{\upsilon}_1 = \upsilon_1$ when $\vartheta_2 = 1$, CP 1's peering decision becomes independent of the existence of CP 2, as it will not obtain any of the inelastic users of CP 2. As a result, the low-value CP 1's peering decision is mostly driven by the *price war* between the APs, similar to monopolistic CP case shown in Figure 2. Based on this benchmark scenario, we will study the impact of 1) CPs' value $\boldsymbol{q}$ and content preservability $\boldsymbol{\delta}$, 2) the market baselines $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, and 3) the user stickiness $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ on the peering equilibria in the following subsections.

[3]In our evaluations, we test all 16 possible peering profiles to see if they are Nash equilibria and in most cases find a unique Nash equilibrium, except for some case of non-existence of Nash equilibrium shown in Figure 7.

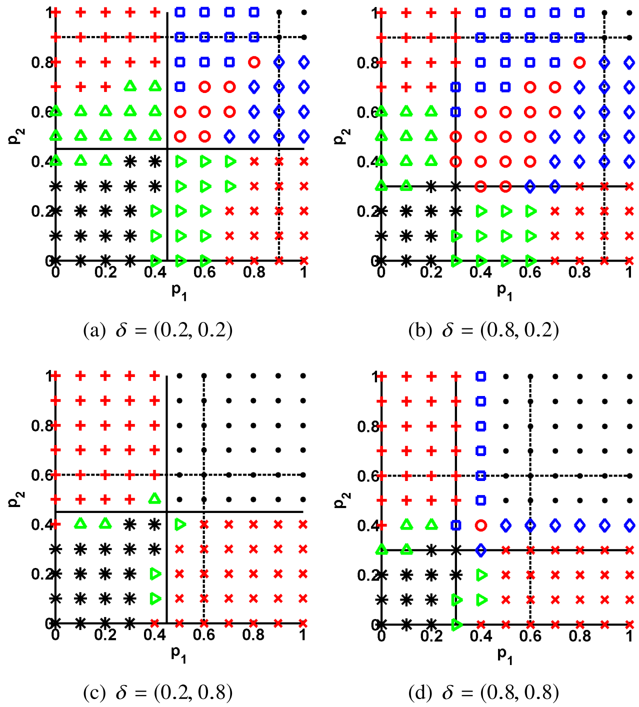


(a) $q = (0.4, 0.8)$ (b) $q = (0.6, 0.8)$ (c) $q = (0.4, 1.0)$ (d) $q = (0.6, 1.0)$

Fig. 5. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\phi} = \boldsymbol{\psi} = \boldsymbol{\delta} = (0.5, 0.5)$.

A. Impact of Value and Preservability

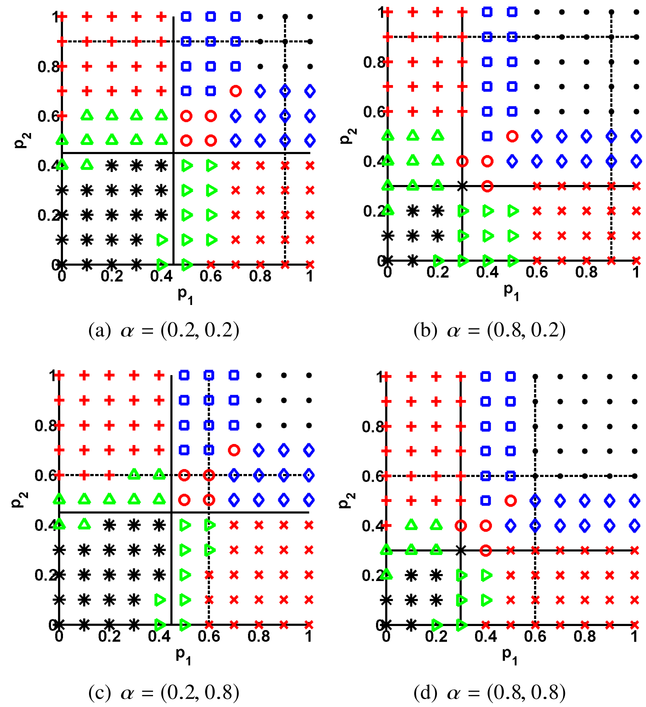
Figure 5 illustrates the impact of value q_i on the equilibria. As q_i scales υ_i linearly, we observe that the increase in q_i also moves CP i's effective VoPP towards a higher value. Although q_i does not affect the effective VoPP of the other CP $\bar{i}$ too much, when q_1 increases from 0.4 to 0.6, there are cases where CP 2 starts to peer with the APs, e.g., $\boldsymbol{p} = (0.5, 0.4)$ in the



(a) $\delta = (0.2, 0.2)$ (b) $\delta = (0.8, 0.2)$

(c) $\delta = (0.2, 0.8)$ (d) $\delta = (0.8, 0.8)$

Fig. 6. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\phi} = \boldsymbol{\psi} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.



(a) $\alpha = (0.2, 0.2)$ (b) $\alpha = (0.8, 0.2)$

(c) $\alpha = (0.2, 0.8)$ (d) $\alpha = (0.8, 0.8)$

Fig. 7. Nash equilibria under complementary duopoly with $\boldsymbol{\beta} = \boldsymbol{\phi} = \boldsymbol{\psi} = \boldsymbol{\delta} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.

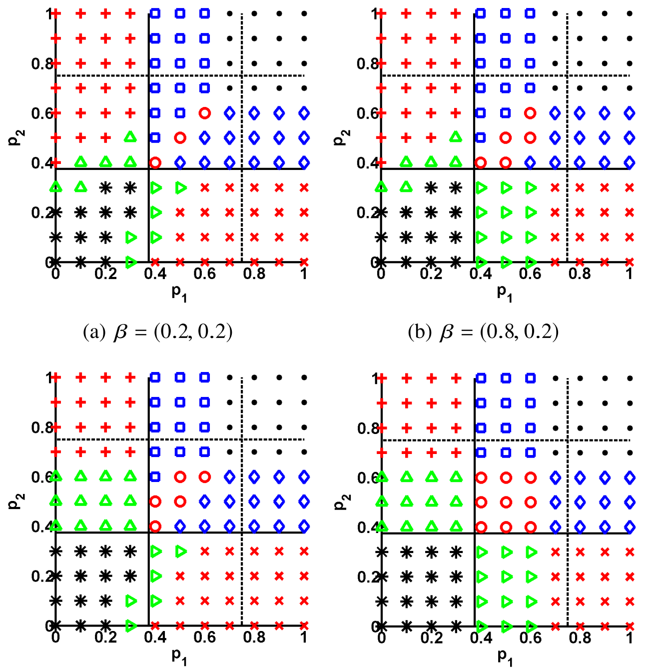
upper sub-figures and $\boldsymbol{p} = (0.5, 0.3)$ and $(0.6, 0.4)$ in the lower sub-figures. We also observe that a CP's effective VoPP with an AP does not decrease monotonically with the other AP's price. For example, in the lower left (right) sub-figure, when $p_j = 0.5$ (0.6) and $p_{\bar{j}}$ decreases from 0.3 (0.5) to 0.2 (0.4), CP 2 will start to peer with AP j. In both cases, we find that either the increase in q_1 or the decrease in p_j attracts CP 1 to peer with AP j, which provides some *peer pressure* for CP 2 to peer with the other AP $\bar{j}$, even CP 2 has already peered with AP j. In contrast with CP 2's behavior, in the left sub-figures, we observe that under $p_j = 0.3$ and $p_{\bar{j}} = 0.5$, when q_2 increases from 0.8 to 1.0, CP 1 starts to de-peer with AP j. We also observe that the effective VoPP of CP 1 for AP j is larger than its intrinsic VoPP υ_1 when AP $\bar{j}$'s price is high, under which CP 2 does not peer with $\bar{j}$. Although $p_j > \upsilon_1$, by peering with AP j, CP 1 can attract some of the non-sticky users from AP $\bar{j}$ so as to recover its peering cost. Both cases show that the low-value CP behave oppositely such that it peers with an AP j if the high-value CP tends not to peer with the other AP $\bar{j}$.

Figure 6 illustrates the impact of content preservability δ_i on the equilibria. As an increase in δ_i reduces υ_i, we observe that the increase in δ_i also moves CP i's effective VoPP towards a lower value. Similarly, although δ_i does not affect the effective VoPP of the other CP $\bar{i}$ too much, when p_j decreases or δ_1 decreases from 0.8 to 0.2, we observe cases where CP 2 starts to peer with AP $\bar{j}$ due to the *peer pressure* of CP 1 peering with AP j, e.g., $\boldsymbol{p} = (0.6, 0.3)$ and $(0.7, 0.4)$ in the upper sub-figures and $\boldsymbol{p} = (0.5, 0.4)$ in the lower sub-figures. Under $p_j = 0.3$ and $p_{\bar{j}} = 0.5$, we observe that when δ_2 increases from 0.2 to 0.8 in the right sub-figures, CP 2 starts to de-peer with AP $\bar{j}$; however, CP 1 starts to peer with AP j. This observation again shows that the low-value CP behave oppositely to the high-vale CP 2.

Lessons Learned: As CP i's intrinsic VoPP υ_i increases with q_i and decreases with δ_i, the increase in q_i or decrease in δ_i moves CP i's effective VoPP of towards a higher value. Although q_i and δ_i do not impact the effective VoPP of the other CP $\bar{i}$ much, the low-value CP's peering with an AP provides some *peering pressure* for the high-value CP to peer with *the other* AP; however, the low-value CP peers with an AP if the high-value CP tends not to peer with the other AP.

B. Impact of User Stickiness

Figure 7 illustrates the impact of the user stickiness α_i of the CPs on the equilibria. Notice that no unique Nash equilibrium exists at $\boldsymbol{p} = (0.3, 0.5)$ and $(0.5, 0.3)$ in the upper-right sub-figure. Similar to the impact of δ_i, as the intrinsic VoPP υ_i decreases with α_i, we observe that the increase in α_i also moves CP i's effective VoPP towards a lower value. Different from the impact of δ_i, we observe that the change in α_1 affects CP 2's effective VoPP with an AP j substantially, especially at a high price $p_{\bar{j}}$ of the other AP; however, the change in α_2 does not change that of CP 1 much. This shows that when the users of the low-value CP become less sticky, it has more incentives to peer with an AP j, resulting in high *peer pressure* to the high-value CP to peer with the *same* AP j, especially when the high-value CP does not peer with the other AP $\bar{j}$ due to its high price $p_{\bar{j}}$. Nevertheless, the change in α_2 only influence CP 2's effective VoPP with an AP j slightly when $p_{\bar{j}}$ is small, very different from the impact of δ_2 on that of CP 2. This is because in that price region, CP 1 cannot afford to peer with AP j and therefore, CP 2 will not lose



(a) β = (0.2, 0.2) (b) β = (0.8, 0.2)

(c) β = (0.2, 0.8) (d) β = (0.8, 0.8)

Fig. 8. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\phi} = \boldsymbol{\psi} = \boldsymbol{\delta} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.

its inelastic users, and therefore, the user stickiness does not impact CP 2 much, unless CP 1 peers with the other AP $\bar{j}$ when $p_{\bar{j}}$ is very low. This is similar to the monopolistic AP scenario where the closed-from of the effective VoPP $\tilde{\upsilon_i}$ is expressed Theorem 8: as CP 1 does not peer with the AP, for CP 2, $\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1})$ does not depend on α_2 and the coefficient $\big(\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}_{-i}=\mathbf{0}\}}\big)$ equals 1, and therefore, its effective VoPP becomes independent of α_2.

Figure 8 illustrates the impact of the user stickiness β_j of the APs on the equilibria. Because any CP's intrinsic VoPP υ_i is not function of β_j, we observe that the APs' user stickiness do not strongly impact the effective VoPP of the CPs. Although an AP j's stickiness β_j does not affect CPs' effective VoPPs with the other AP $\bar{j}$, it does affect those with AP j itself. As shown in all previous plots, a CP's effective VoPP with an AP j decreases as the competitor AP's price $p_{\bar{j}}$ decreases; however, by comparing cases under different β_j, we observe that this decreasing trend is more prominent when β_j is small. In particular, when $\beta_j = 0.8$, the AP becomes like a monopoly over its baseline market share of users and therefore, CPs' effective VoPPs are very close to their intrinsic VoPPs υ_i and do not get affected by the competitor's price $p_{\bar{j}}$ much. Similar observations can also be made in the monopolistic CP scenario in Figure 2. In general, our observations imply that with low user stickiness, an AP is more susceptible to market competition with its competitors.

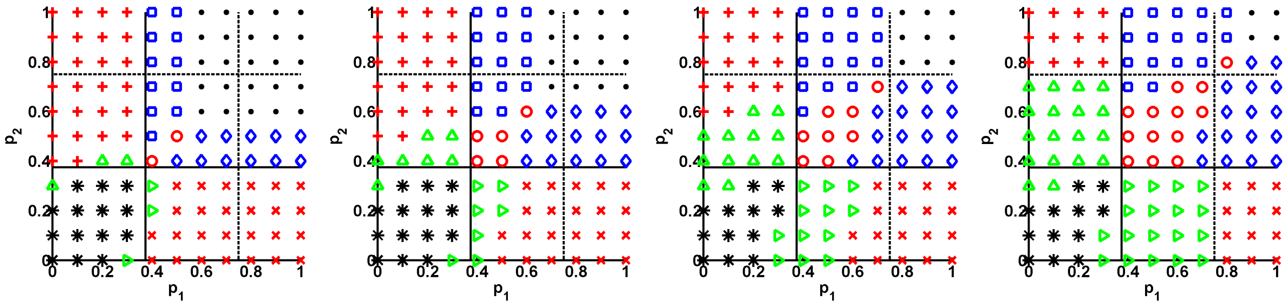
Lessons Learned: As CP i's intrinsic VoPP υ_i decreases with α_i, the increase in α_i moves the effective VoPP of CP i towards a lower value. The low-value CP's peering decision with an AP provides *peering pressure* for the high-value CP to peer with *the same* AP; however, it does not get affected by the user stickiness α_i of the high-value CP. The user stickiness β_j of an AP j mostly affect CPs' effective VoPPs with itself and larger user stickiness will make the AP more like a monopoly over its baseline market share of users, under which any CP's effective VoPP is close to its intrinsic VoPP.

C. Impact of Market Baseline

Figure 9 illustrates the impact of the market baseline ϕ_i of the CPs on the equilibria. We observe that as the baseline market shares shift more towards the low-value CP 1 from the left to the right sub-figures, the effective VoPP of the high-value CP 2 moves towards a higher value prominently; however, that of CP 1 changes slightly, i.e., CP 1 de-peers at $\boldsymbol{p} = (0.3, 0.1)$ and $(0.1, 0.3)$ when ϕ_1 increases from 0.4 to 0.6. This observation shows that when the low-value CP 1 has a larger baseline market share, it will have fewer incentives to peer with the expensive AP, as peering with the cheap AP can already keep a substantial market share. In general, we observe that the low-value CP 1's effective VoPP is very close to its intrinsic VoPP υ_1 as explained before. At the price region of CP 2's effective VoPP, because the low-value CP 1 cannot afford to peer with any AP, CP 2 can attract CP 1's non-sticky users, even if it only peers with one of the APs. As CP 1's market baseline increases, this gain from CP 1's non-sticky users also becomes larger, which increases CP 2's effective VoPP consequently. As a result, although we observe that CP 2's effective VoPP with a particular AP could be much smaller than its intrinsic VoPP υ_2 when it has a larger baseline than CP 1 in the left sub-figures, it becomes clearly larger than $\upsilon_2 = 0.75$ in the right-most sub-figure when $\phi_1 = 0.8$ and the competitor AP's price is not lower than 0.8. This shows even both APs' prices are higher than υ_2, CP 2 is better off by peering with an AP and compensated by the elastic market share of CP 1. A similar observation can be made in the monopolistic AP scenario: given CP 1 does not peer, i.e., $\vartheta_1 = 0$, CP 2's effective VoPP satisfies $\tilde{\upsilon_2} = \left(1 - \dfrac{\phi_2}{1-\alpha_1\phi_1}\delta_2\right) q_2$, which is no smaller than υ_2.

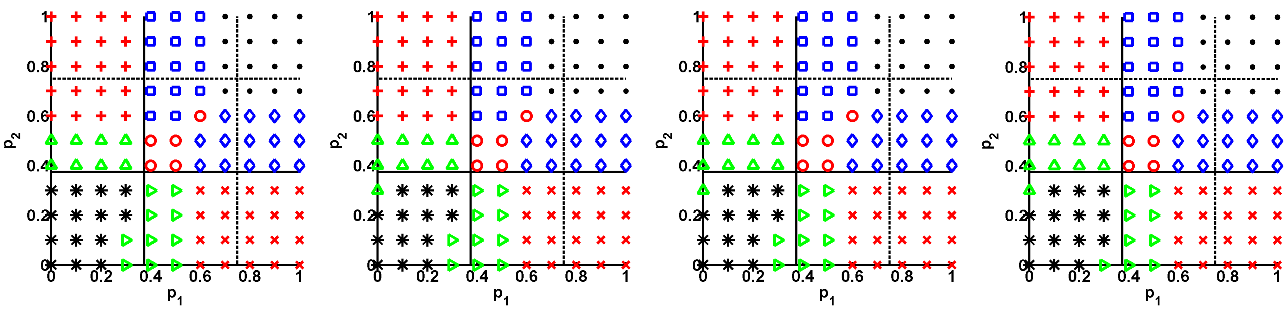
Figure 10 illustrates the impact of the market baseline ψ_i of the APs on the equilibria. We observe that as the baseline market shares shift towards AP 1 from the left to the right sub-figures, the effective VoPP of the high-value CP 2 does not change; however, that of CP 1 changes slightly: CP 1 starts to peer with AP 1 at $\boldsymbol{p} = (0.3, 0.1)$ when ψ_1 increases from 0.3 to 0.4 and starts to de-peer with AP 2 at $\boldsymbol{p} = (0, 0.3)$ when ψ_1 increases from 0.1 to 0.2. By comparing the result of Corollary 4, which shows that the effective VoPP does not depend on APs' baselines under a monopolistic CP, this observation shows that with the existence of a high-value CP 2, which will take CP 1's non-sticky users if it does not peer, the low-value CP 1 will have more incentives to peer with an AP j if it has a higher market baseline ψ_j.

Lessons Learned: The high-value CP's peering decision is greatly influenced by the baseline market share distribution of the CPs, but not that of the APs. In particular, the decrease in its market baseline moves its effective VoPP towards a higher value, which can possibly be greater than its intrinsic VoPP. The low-value CP's peering decision, however, is only slightly affected by the providers' market baselines: its effective VoPP



(a) ϕ = (0.2, 0.8) (b) ϕ = (0.4, 0.6) (c) ϕ = (0.6, 0.4) (d) ϕ = (0.8, 0.2)

Fig. 9. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\psi} = \boldsymbol{\delta} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.



(a) ψ = (0.1, 0.9) (b) ψ = (0.2, 0.8) (c) ψ = (0.3, 0.7) (d) ψ = (0.4, 0.6)

Fig. 10. Nash equilibria under complementary duopoly with $\boldsymbol{\alpha} = \boldsymbol{\beta} = \boldsymbol{\phi} = \boldsymbol{\delta} = (0.5, 0.5)$ and $\boldsymbol{q} = (0.5, 1.0)$.

with an AP j will increase if its market baseline decreases or that of AP j, i.e., ψ_j, increases.

VI. Discussions and Conclusions

Based on our qualitative results, we understand the driving forces behind premium peering in practice as follows. As 77-95% of broadband users are sticky [17], APs have high values for β_j and could charge a CP up to its intrinsic VoPP. By delivering video traffic, Level 3 serves Netflix as a CDN; however, its demand is very elastic, i.e., α_i is close to zero, as Netflix uses other CDNs, e.g., Limelight. As a result, its intrinsic VoPP υ_i is close to its CDN price q_i [39]. Level 3 agreed on the premium peering because Comcast charged as low as IP transit prices, which are lower than those of CDNs. As a quality-sensitive video streaming provider, Netflix has a high value q_i and low content preservability δ_i, and thus a much higher intrinsic VoPP υ_i than those of the CDNs. Although it has a 38% market share [17], this high-value CP entered into premium peering with many APs, e.g., Comcast and Verizon, due to the induced *peer pressure* when low-value competitors, including APs' own content services, use premium peering. Furthermore, as the user stickiness of CPs are estimated between 36-80% [17], there are still substantial market share of non-sticky users for Netflix to compete for.

We foresee that APs might increase the peering price and want to further differentiate its services for higher charges, because even with a small baseline ψ_j, an AP could behave like a monopoly over its users given their high stickiness. If the AP market is competitive, under which users are non-sticky, APs will enter into a *price war*, resulting in low competitive market prices. Thus, from a regulatory perspective, it will be more important to provide service transparency and alternatives to users, i.e., to reduce β_j of the APs, rather than regulating the peering mechanisms and prices. As the peering quality also affects APs' user satisfaction, in theory, it is possible that competitive APs will pay monopolistic CPs for premium services so as to compete for end-users, which can be analyzed similarly under our model. However, regulating monopolistic CPs might be beyond the FCC's jurisdiction.

In conclusion, whether an AP's premium peering will be used mainly depends on its user stickiness and price, while its market baseline plays a minor role; the content value and preservability of a CP mainly determine its intrinsic VoPP, which fundamentally determines the CP's peering decision. In particular, high-value CPs have *peer pressure* when low-value CPs peer; however, low-value CPs behave oppositely. The peering decisions of the low- and high-value CPs are substantially influenced by their user stickiness and baseline market shares, respectively, but not vice versa.

Appendix

Proof of Theorem 1: First, one can easily verify that when $\Theta = \mathbf{0}$ or $\mathbf{1}$, the market share of each pair (i, j) of providers equals $X_{ij} = \phi_i \psi_j X$. When $\theta_{ij} = 0$, any user of (i, j) will stick with the same pair of providers with probability $\alpha_i \beta_j$ by Assumption 2. With probability $\alpha_i \bar{\beta}_j$, a user is not sticky to AP j; however, if $\boldsymbol{\theta_i} = \mathbf{0}$, which implies that none of the peering link with CP i is paid peering, then the choice set $\mathcal{O}$ for the user will be empty, and therefore, she will still stick with (i, j). By the same reason, if $\boldsymbol{\vartheta_j} = \mathbf{0}$ (or $\Theta = \mathbf{0}$), none of the peering link with AP j (or any of the peering links) is paid peering, and therefore, $\bar{\alpha}_i\beta_j$ (or $\bar{\alpha}_i\bar{\beta}_j$) percentage of the users will stick with (i, j). When we add the four cases in Assumption 2, we obtain that for $\theta_{ij} = 0$, $\rho_{ij} = \alpha_i\beta_j + \alpha_i\bar{\beta}_j\mathbf{1}_{\{\boldsymbol{\theta}_i=\mathbf{0}\}} + \bar{\alpha}_i\beta_j\mathbf{1}_{\{\boldsymbol{\vartheta}_j=\mathbf{0}\}} + \bar{\alpha}_i\bar{\beta}_j\mathbf{1}_{\{\Theta=\mathbf{0}\}}$. When $\theta_{ij} = 1$, all the original $\phi_i\psi_jX$ number of users will not switch providers. There are three other types of users under public peering that might switch to the pair (i, j): the users of (i, j'), (i', j) and (i', j'). The total number of potential users from (i, j') equals $\sum_{j\in\mathcal{M}}(1-\theta_{ij})\alpha_i\bar{\beta}_j\phi_i\psi_jX$, because they sticky with CP i but not sticky with AP j under public peering links. Because each user will join AP j with probability $\frac{\psi_j}{\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}}$ by Assumption 1, the total number of users that are sticky with CP i and shift from AP j' to j equals $\frac{\psi_j}{\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}}\sum_{j\in\mathcal{M}}\bar{\theta}_{ij}\alpha_i\bar{\beta}_j\phi_i\psi_jX$, which can be expressed as $\phi_i\psi_jX\frac{\alpha_i\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}}$ in a vector form. By the same logic, we derive the number of shifted users to (i, j) from the other two types as $\phi_i\psi_jX\frac{\beta_j\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}}{\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}}$ and $\phi_i\psi_jX\frac{\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}{}^T\bar{\Theta}\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\phi}^T\bar{\Theta}\boldsymbol{\psi}}$. Finally, by adding the users from all the above cases, we obtain

$$\rho_{ij} = 1 + \frac{\alpha_i\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}} + \frac{\beta_j\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}}{\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}} + \frac{\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}{}^T\bar{\Theta}\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\phi}^T\bar{\Theta}\boldsymbol{\psi}}. \qquad \blacksquare$$

Proof of Corollary 1: Given $\tilde{\Theta} \geq \Theta$, we have $\tilde{\boldsymbol{\theta}}_i \geq \boldsymbol{\theta}_i$, $\tilde{\boldsymbol{\vartheta}}_j \geq \boldsymbol{\vartheta}_j$, which implies that $\mathbf{1}_{\{\tilde{\boldsymbol{\theta}}_i=\mathbf{0}\}} \leq \mathbf{1}_{\{\boldsymbol{\theta}_i=\mathbf{0}\}}$, $\mathbf{1}_{\{\tilde{\boldsymbol{\vartheta}}_j=\mathbf{0}\}} \leq \mathbf{1}_{\{\boldsymbol{\vartheta}_j=\mathbf{0}\}}$ and $\mathbf{1}_{\{\tilde{\Theta}=\mathbf{0}\}} \leq \mathbf{1}_{\{\Theta=\mathbf{0}\}}$. By Theorem 1, we deduce that $X_{ij}(\tilde{\Theta}) \leq X_{ij}(\Theta)$ if $\theta_{ij} = 0$. We also deduce that the terms $\frac{\alpha_i\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\bar{\boldsymbol{\theta}}_i\boldsymbol{\psi}}$, $\frac{\beta_j\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}}{\bar{\boldsymbol{\vartheta}}_j\boldsymbol{\phi}}$ and $\frac{\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}{}^T\bar{\Theta}\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\phi}^T\bar{\Theta}\boldsymbol{\psi}}$ are non-increasing when the peering matrix changes from Θ to $\tilde{\Theta}$, because the numerators are non-increasing and the denominators are non-decreasing, which implies that $X_{ij}(\tilde{\Theta}) \leq X_{ij}(\Theta)$ if $\theta_{ij} = 1$. $\blacksquare$

Proof of Corollary 2: The additivity property holds because users choose providers based on the linear proportional form of Assumption 1, which induces the market shares of the providers in terms of linear functions in Theorem 1. When providers have the same stickiness, i.e., α_i or β_j, and peering profiles, i.e., $\boldsymbol{\theta}_i$ or $\boldsymbol{\vartheta}_j$, the closed-form market share expressions keep the same form in terms of $\rho_{ij}(\Theta)$, while the baselines of the providers, i.e., ϕ_i or ψ_j, are aggregated. $\blacksquare$

Proof of Theorem 2: Since there are no alternative providers, none of the X users will be lost under public peering. By the definition of utilities in Equation (2), we obtain (3). The CP will use paid peering only if its derived utility $(q-p)X$ is no smaller than that under public peering, i.e., δqX, which implies the condition $p \leq (1-\delta)q$. Without cooperation, the CP and AP obtains utilities δqX and 0, respectively, under public peering. The Shapley value is the solution of the balanced contribution condition $U - \delta qX = R - 0$ and the efficiency condition $U + R = qX$. The Nash bargaining solution, on the other hand, solve the optimization problem:

$$\text{Maximize} \quad [(q-p)X - \delta qX][pX - 0], \quad \text{s.t.} \quad p \leq (1-\delta)q.$$

The solution of both turns out to be $U = \frac{1}{2}(1+\delta)qX$ and $R = \frac{1}{2}(1-\delta)qX$, which implies that the corresponding price satisfies $p = \frac{1}{2}(1-\delta)q$. $\blacksquare$

Proof of Theorem 3: By applying Theorem 1, we can derive $X_j(\boldsymbol{\theta}) = \rho_j(\boldsymbol{\theta})\psi_jX$, where

$$\rho_j(\boldsymbol{\theta}) = \begin{cases} \beta_j + \bar{\beta}_j\mathbf{1}_{\{\boldsymbol{\theta}=\mathbf{0}\}} & \text{if} \quad \theta_j = 0, \\ 1 + \dfrac{\bar{\boldsymbol{\theta}}\boldsymbol{\psi}_{\bar{\boldsymbol{\beta}}}}{\boldsymbol{\theta}\boldsymbol{\psi}} = 1 + \dfrac{\bar{\boldsymbol{\theta}}(\bar{\boldsymbol{\beta}}\circ\boldsymbol{\psi})}{\boldsymbol{\theta}\boldsymbol{\psi}} & \text{if} \quad \theta_j = 1. \end{cases}$$

From the above equation, we deduce that

$$\theta_j\rho_j(\boldsymbol{\theta}) = \theta_j\left[1 + \frac{\bar{\boldsymbol{\theta}}(\bar{\boldsymbol{\beta}}\circ\boldsymbol{\psi})}{\boldsymbol{\theta}\boldsymbol{\psi}}\right] = \frac{\theta_j}{\boldsymbol{\theta}\boldsymbol{\psi}}[\boldsymbol{\theta}\boldsymbol{\psi} + \bar{\boldsymbol{\theta}}(\bar{\boldsymbol{\beta}}\circ\boldsymbol{\psi})].$$

Because $\boldsymbol{\theta}\boldsymbol{\psi} + \bar{\boldsymbol{\theta}}(\bar{\boldsymbol{\beta}}\circ\boldsymbol{\psi}) + \bar{\boldsymbol{\theta}}(\boldsymbol{\beta}\circ\boldsymbol{\psi}) = 1$, we have

$$\theta_j\rho_j(\boldsymbol{\theta}) = \frac{\theta_j}{\boldsymbol{\theta}\boldsymbol{\psi}}[1 - \bar{\boldsymbol{\theta}}(\boldsymbol{\beta}\circ\boldsymbol{\psi})] = \frac{\theta_j}{\boldsymbol{\theta}\boldsymbol{\psi}}[1 - (\bar{\boldsymbol{\theta}}\circ\boldsymbol{\beta})\boldsymbol{\psi}].$$

Thus, if $\theta_j = 1$, then

$$R_j(\boldsymbol{\theta}) = p_jX_j(\boldsymbol{\theta}) = p_j\rho_j(\boldsymbol{\theta})\psi_jX = p_j\frac{1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})}{\boldsymbol{\theta}\boldsymbol{\psi}}\psi_jX.$$

The total payment $R(\boldsymbol{\theta}) \triangleq \sum_{j\in\mathcal{M}}R_j(\boldsymbol{\theta})$ becomes

$$\sum_{j\in\mathcal{M}} p_j\theta_j\frac{1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})}{\boldsymbol{\theta}\boldsymbol{\psi}}\psi_jX = \frac{[1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]X}{\boldsymbol{\theta}\boldsymbol{\psi}}(\boldsymbol{p}\circ\boldsymbol{\theta})\boldsymbol{\psi}.$$

When $\boldsymbol{\theta} = \mathbf{0}$, all users are under public peering and therefore, the CP's utility equals δqX. When $\boldsymbol{\theta} \neq \mathbf{0}$, $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})$ proportion of the users will be sticky to their APs under public peering, and therefore, the users will generate revenue:

$$[1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]qX + \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\delta qX = \left[q - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon\right]X. \qquad \blacksquare$$

Proof of Corollary 3: Theorem 3 shows that under any fixed $\boldsymbol{\theta}$, $U(\boldsymbol{\theta})$ is a continuous, non-increasing and linear function in the prices $\boldsymbol{p}$. $U^*(\boldsymbol{p})$ is the maximum of all these linear functions, and therefore, is a convex function in $\boldsymbol{p}$. Again by Theorem 3, $U(\boldsymbol{\theta}) = \delta qX$ if $\boldsymbol{\theta} = \mathbf{0}$. When $p_j = \upsilon$ for all $j \in \mathcal{M}$, for any $\boldsymbol{\theta} \neq \mathbf{0}$, the average price $(\boldsymbol{p}\circ\boldsymbol{\theta})\boldsymbol{\psi}/\boldsymbol{\theta}\boldsymbol{\psi}$ equals υ, and therefore, $R(\boldsymbol{\theta}) = \left[1 - \bar{\boldsymbol{\theta}}(\boldsymbol{\beta}\circ\boldsymbol{\psi})\right]\upsilon$. By substituting it into $U(\boldsymbol{\theta})$, we obtain $U(\boldsymbol{\theta}) = \delta qX$. $\blacksquare$

Proof of Theorem 4: By Corollary 3, $U^*(\boldsymbol{\upsilon}) = U(\mathbf{0})$ and $U^*(\boldsymbol{p})$ is non-increasing in $\boldsymbol{p}$, therefore, $U^*(\boldsymbol{p}) \leq U(\mathbf{0})$ for all $\boldsymbol{p} \geq \boldsymbol{\upsilon}$. Since $U(\mathbf{0})$ is always achievable, $U^*(\boldsymbol{p}) = U(\mathbf{0})$. To show $\theta_j^* = 0$ for all $p_j > \upsilon$, we first show that $(\boldsymbol{p}\circ\boldsymbol{\theta}^*)/\boldsymbol{\theta}^*\boldsymbol{\psi} < \upsilon$. Because otherwise, by Theorem 3, we deduce that $U(\boldsymbol{\theta}^*) < \delta qX = U(\mathbf{0})$, which contradicts the optimality of $\boldsymbol{\theta}^*$. Second, we show that for all paid-peering APs with prices $p_j > \upsilon$, we can increase the CP's utility by setting $\theta_j = 0$ for all of them. In the worst case when all their users are sticky, i.e., $\beta_j = 1$, and we lose υ per-user, which is less than their average price. However, if some users are non-sticky, they will move to APs with prices lower than υ, which further will increase the CP's utility. $\blacksquare$

Proof of Theorem 5: When $\tilde{\boldsymbol{\theta}} = \mathbf{0}$, by Theorem 3, $U(\tilde{\boldsymbol{\theta}}) = \delta qX$ and $U(\boldsymbol{\theta}) = \{q - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon - (1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta}))p_j\}X$. Thus, $U(\boldsymbol{\theta}) \geq U(\tilde{\boldsymbol{\theta}})$ is equivalent to $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon + (1-\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta}))p_j \leq \upsilon$, which yields $p_j \leq \upsilon$. Again, by Theorem 3, $U(\boldsymbol{\theta}) \geq U(\tilde{\boldsymbol{\theta}})$ is equivalent to $\Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})\upsilon X + R(\boldsymbol{\theta}) \leq \Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}})\upsilon X + R(\tilde{\boldsymbol{\theta}})$. When $\tilde{\boldsymbol{\theta}} \neq \mathbf{0}$, $\Psi_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\theta}}) = \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta}) + \beta_j\psi_j$, which implies that

$R(\boldsymbol{\theta}) \leq \beta_j \psi_j v X + R(\tilde{\boldsymbol{\theta}})$. By dividing $[1 - \Psi_{\boldsymbol{\beta}}(\boldsymbol{\theta})]X$ on both sides, we obtain the result. ■

Proof of Corollary 4: Given $\tilde{\boldsymbol{\theta}} = \mathbf{0}$, Theorem 5 shows that $\theta_j = 1$ is optimal if and only if $\theta_j \leq v$. Given $\tilde{\boldsymbol{\theta}} = (0, 1)$ and $\tilde{\boldsymbol{\theta}} = (1, 0)$, Theorem 5 shows that $\theta_j = 1$ is optimal if and only if $p_j \leq \beta_j v + \bar{\beta}_j p_{\bar{j}}$. By combining both conditions, we obtain that $\theta_j = 1$ is optimal if and only if $p_j \leq \min\{\beta_j v + \bar{\beta}_j p_{\bar{j}}, v\} = \beta_j v + \bar{\beta}_j \min\{p_{\bar{j}}, v\}$. ■

Proof of Theorem 6: We prove the result by showing that the CP's utility can increase if we swap the peering strategies θ_j and θ_m, and therefore, $\boldsymbol{\theta}$ is not optimal. We only needs to focus on the non-trivial case where $\boldsymbol{\theta}^* \neq \mathbf{0}$. Based on Equation (3), the optimal strategy $\boldsymbol{\theta}^*$ solves

$$\text{Minimize} \quad (\bar{\boldsymbol{\theta}} \circ \boldsymbol{\beta})\boldsymbol{\psi} v + \Big[1 - (\bar{\boldsymbol{\theta}} \circ \boldsymbol{\beta})\boldsymbol{\psi}\Big] \frac{(\boldsymbol{p} \circ \boldsymbol{\theta})\boldsymbol{\psi}}{\boldsymbol{\theta}\boldsymbol{\psi}}.$$

Because $p_j \psi_j > p_m \psi_m$ and $\beta_j \psi_j < \beta_m \psi_m$, swapping the peering decisions will reduce the average price $(\boldsymbol{p} \circ \boldsymbol{\theta})\boldsymbol{\psi}/\boldsymbol{\theta}\boldsymbol{\psi}$ and the number of non-sticky users $(\bar{\boldsymbol{\theta}} \circ \boldsymbol{\beta})\boldsymbol{\psi}$, and therefore, will increase the CP's utility. ■

Proof of Theorem 7: By applying Theorem 1, we can derive $X_i(\boldsymbol{\vartheta}) = \rho_i(\boldsymbol{\vartheta})\phi_i X$, where

$$\rho_i(\boldsymbol{\vartheta}) = \begin{cases} \alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}=\mathbf{0}\}} & \text{if} \quad \vartheta_i = 0, \\ 1 + \dfrac{\bar{\boldsymbol{\vartheta}}\boldsymbol{\phi}_{\bar{\boldsymbol{\alpha}}}}{\boldsymbol{\vartheta}\boldsymbol{\phi}} = 1 + \dfrac{\bar{\boldsymbol{\vartheta}}(\bar{\boldsymbol{\alpha}} \circ \boldsymbol{\phi})}{\boldsymbol{\vartheta}\boldsymbol{\phi}} & \text{if} \quad \vartheta_i = 1. \end{cases}$$

By the same logic used in Theorem 3, we deduce that

$$\vartheta_i \rho_i(\boldsymbol{\vartheta}) = \frac{\vartheta_i}{\boldsymbol{\vartheta}\boldsymbol{\phi}}[1 - (\bar{\boldsymbol{\vartheta}} \circ \boldsymbol{\alpha})\boldsymbol{\phi}] = \frac{\vartheta_i}{\boldsymbol{\vartheta}\boldsymbol{\phi}}[1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})].$$

By using the definition of $U_i(\boldsymbol{\vartheta})$ in (2), we deduce (8). Furthermore, $R(\boldsymbol{\vartheta}) = p \sum_{i \in \mathcal{N}} \vartheta_i \rho_i(\boldsymbol{\vartheta})\phi_i X = [1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta})]pX$. ■

Proof of Theorem 8: Given any $\boldsymbol{\vartheta}_{-i}$, by Theorem 7, we know that $U_i(\vartheta_i = 0; \boldsymbol{\vartheta}_{-i}) = \big[\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}_{-i}=\mathbf{0}\}}\big] q_i \delta_i \phi_i X$ and $U_i(\vartheta_i = 1; \boldsymbol{\vartheta}_{-i}) = \dfrac{1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1})}{\boldsymbol{\vartheta}_{i=1}\boldsymbol{\phi}}(q_i - p)\phi_i X$. Thus, CP i uses $\vartheta_i = 1$ if and only if $U_i(\vartheta_i = 1; \boldsymbol{\vartheta}_{-i}) \geq U_i(\vartheta_i = 0; \boldsymbol{\vartheta}_{-i})$, which implies the condition of Nash equilibrium as

$$\big[\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}_{-i}=\mathbf{0}\}}\big] \frac{\boldsymbol{\vartheta}_{i=1}\boldsymbol{\phi}}{1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1})} \delta_i \leq 1 - p/q_i,$$

or equivalently, $p \leq \tilde{v}_i(\boldsymbol{\vartheta}_{-i})$. Finally, when $\boldsymbol{\vartheta}_{-i} = \mathbf{1}$, we have $\boldsymbol{\vartheta}_{i=1}\boldsymbol{\phi} = \mathbf{1}\boldsymbol{\phi} = 1$ and $\Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1}) = \Phi_{\boldsymbol{\alpha}}(\mathbf{1}) = 0$, and therefore, $\tilde{v}_i(\boldsymbol{\vartheta}_{-i}) = (1 - \alpha_i \delta_i) q_i = v_i$; when $\boldsymbol{\alpha} = \mathbf{1}$, $\big[\alpha_i + \bar{\alpha}_i \mathbf{1}_{\{\boldsymbol{\vartheta}_{-i}=\mathbf{0}\}}\big] = 1$ and $\boldsymbol{\vartheta}_{i=1}\boldsymbol{\phi} = 1 - \Phi_{\boldsymbol{\alpha}}(\boldsymbol{\vartheta}_{i=1})$, which implies that $\tilde{v}_i(\boldsymbol{\vartheta}_{-i}) = (1 - \delta_i) q_i = v_i$ for all $\boldsymbol{\vartheta}_{-i}$. ■

Proof of Theorem 9: Because $1 - \alpha_1 \alpha_2 = \alpha_1 \bar{\alpha}_2 + \bar{\alpha}_1 \alpha_2 + \bar{\alpha}_1 \bar{\alpha}_2$, we know $\gamma_1 + \gamma_2 = \dfrac{\alpha_1 \bar{\alpha}_2 + \bar{\alpha}_1 \alpha_2}{\alpha_1 \bar{\alpha}_2 + \bar{\alpha}_1 \alpha_2 + \bar{\alpha}_1 \bar{\alpha}_2} < 1$. Because $\phi_1 + \phi_2 = 1$, we cannot have a case that $\phi_1 \leq \gamma_1$ and $\phi_2 \leq \gamma_2$, and therefore, the mutually exclusive cases (a)-(c) covers all the cases for $\alpha_1 \alpha_2 < 1$. Because $\alpha_1 \delta_1 \geq \alpha_2 \delta_2$ and $q_1 = q_2$, we deduce that $(1 - \alpha_1 \delta_1) q_1 = v_1 \leq v_2 = (1 - \alpha_2 \delta_2) q_2$. For $N = 2$, let $\bar{i}$ denote the CP other than i. By definition, $\hat{v}_i = \left(1 - \dfrac{\phi_i}{1 - \alpha_{\bar{i}} \phi_{\bar{i}}} \delta_i\right) q_i$, and therefore, $(1 - \alpha_i \delta_i) q_i = v_i \geq \hat{v}_i$ is equivalent to $\alpha_i \leq \dfrac{\phi_i}{1 - \alpha_{\bar{i}} \phi_{\bar{i}}}$ or $\alpha_i \leq \dfrac{\phi_i}{1 - \alpha_{\bar{i}}(1 - \phi_i)}$, which is equivalent to $\phi_i \geq \dfrac{\alpha_i \bar{\alpha}_{\bar{i}}}{1 - \alpha_i \alpha_{\bar{i}}} = \gamma_i$. Thus, $\phi_i \geq \gamma_i$ if and only if $v_i \geq \hat{v}_i$ for $i = 1, 2$. Under case (a), we can deduce that $\hat{v}_1 < v_1 \leq v_2 < \hat{v}_2$, which implies $\min\{v_2, \hat{v_2}\} \geq \max\{v_1, \hat{v_1}\}$. Under case (b), we can deduce that either $\hat{v}_1 \leq v_1 \leq \hat{v}_2 \leq v_2$ or $\max\{\hat{v}_2, \hat{v}_1\} \leq v_1 \leq v_2$, and in the latter case when $p \in (\max\{\hat{v}_2, \hat{v}_1\}, v_1) \neq \emptyset$, it satisfies both $p \geq \max\{\hat{v}_2, \hat{v}_1\}$ and $p \leq \min\{v_1, v_2\}$ and therefore, both (0, 0) and (1, 1) are Nash equilibria. When $\sqrt{\bar{\alpha}_1}\phi_1 < \sqrt{\bar{\alpha}_2}\phi_2$, we deduce that $\hat{v}_2 < \hat{v}_1$, and under case (c), we can deduce that $\hat{v}_2 < v_2$ and $v_1 < \hat{v}_1$. Combining both, we obtain that $\max\{v_1, \hat{v_2}\} < \min\{v_2, \hat{v_1}\}$, and when $p \in \big(\max\{v_1, \hat{v}_2\}, \min\{\hat{v}_1, v_2\}\big) \neq \emptyset$, one can verify that none of the strategy profiles forms a Nash equilibrium. ■

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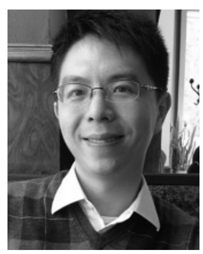

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