Regarding the Optimality of Speedup Bounds of Mixed-Criticality Schedulability Tests

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1 Introduction

Much existing research on Mixed-Criticality (MC) scheduling (see [7] for a review) has focused on dealing with the Vestal model [15], where different WCET estimations of a single piece of code are provided. This is typically a consequence of different tools for determining worst case execution time (WCET) bounds being more or less conservative than each other. It is known [1] that mixed criticality (MC) scheduling under such model is highly intractable, such that polynomial-time optimal solution is impossible unless \( P = NP \). As a result, speedup bound is widely used in MC scheduling for measuring how close to optimal is a given schedulability analysis.

- A schedulability test \( A \) has speedup factor of \( s(s \geq 1) \), if any task set that is schedulable by any algorithm on a given platform with processing speed of 1, it will be deemed schedulable by Test \( A \) upon a platform that is \( s \) times as fast.

Of course, when deriving MC schedulers and associated schedulability tests, one of the goals is to identify/prove a relative small speedup bound (that is closer to 1). A minimum possible speedup is often presented as the “optimal speedup bound” of a given MC scheduling problem. However, we would like to point out that:

- Optimality of scheduler should not be derived against optimal speedup bounds.

2 Non-Optimal Schedulers with Optimal Speedup Bounds

For scheduling (dual-criticality) Vestal job set on a uniprocessor platform, it has been shown [2] that OCBP algorithm (following the idea of Audsley’s priority assignment mechanism) has an optimal speedup bound of \( (\sqrt{5} - 1)/2 \). However, several algorithms has been identified to strictly dominate OCBP; e.g., Lazy Priority Adjustment [10], LE-EDF [12] [11] — with the same speedup bound and at all time, better schedulability.

Similar results can be observed when we consider the scheduling of Vestal task set as well. It has been shown that 4/3 is the best speedup that any non-clairvoyant scheduler can achieve. Upon proposing a speedup-optimal uniprocessor scheduler named EDF-VD [3], improvements on the schedulability can still be made, e.g., [9] [8]. As
for the multiprocessor case, it is proved [4] that both MC-Fluid [13] and MCF [4] achieve the optimal speedup of $4/3$. However, MCF is a simplified version of (and is dominated by) MC-Fluid. Moreover, improvements on schedulability can be further made to MC-Fluid [14].

3 Speedup over Non-Clairvoyance?

When deriving speedup bounds, in most of the existing works of the community, the proposed algorithm is compared with a clairvoyant optimal scheduler, and adapts the necessary conditions for MC schedulability. This may not be a very fair way of comparison, since the penalty for unawareness of the future is applied into the speedup bounds. Following the varying-speed MC model [6] [5], we have identified an online optimal\(^1\) scheduler in [11] that has a speedup factor significantly greater than 1 when comparing to an optimal clairvoyant algorithm. However, such a speedup factor only reflects the price one must pay for not knowing the future (or the difficulty of the scheduling problem itself) — it has nothing to do with the MC scheduler design any more. In other words, most existing speedup bounds may only be capturing the gap between clairvoyance and non-clairvoyance.

Since MC schedulability analysis is for off-line verification of correctness of real-time systems, all possible scenarios should be taken into consideration (which is non-clairvoyance). We believe speedup results comparing to optimal non-clairvoyance schedule may be worth investigating for MC systems.

References


\(^1\)If an on-line optimal scheduling strategy fails to maintain correctness for a given MC instance $I$, no non-clairvoyant algorithm can ensure correctness for $I$ (without making lucky guesses to the future).