Determination of Current Distribution in EM Gun Armature by Least Squares Fitting of $\dot{B}$ Coil Voltage

KENNETH K. COBB

Abstract—The electromagnetic (EM) plasma rail-gun current distribution was determined by fitting (using the method of least squares) the voltage induced on small induction ($\dot{B}$) coils to a derived function model. The voltage function model was derived using the Biot–Savart equation. The model was derived assuming the current flowed in sheets perpendicular to the rails. The sheets of currents varied in time and in the rail direction but were assumed to be constant (at a given time) perpendicular to the bore direction. The plasma current distribution in the bore direction, the initial length, and expansion characteristics of the plasma were determined from $\dot{B}$ coil voltage measurements taken at short time increments. Fitted parameters correlate well with measurements taken by other sensors.

Nomenclature

A_x	Coefficient in plasma distribution function; see (12), (13), and (17).
$\vec{A}_c$	Vector giving orientation of coil and area.
A_c	Area of the coil.
$A_b(t)$	Distance along the x axis from coil to base of the plasma.
$A_{bi}(t)$	Distance along the x axis from coil to base of ith plasma element.
$\vec{B}_e$	Magnetic field at center of coil due to current element.
C_e	Coefficient defined in (10).
C_I	Voltage coefficient (15)
C_{Ii}	Voltage segment coefficient defined in (17).
$A'_{bi}(t)$	Derivative of $A_{bi}(t)$ with respect to time.
D_c	Diameter of coil.
A_{b0}	Initial base of the plasma armature and the base of segment #1.
A_{b0i}	Initial base of ith segment.
a_i	Difference between acceleration of i segment base and projectile.
γ	Difference between acceleration of plasma base and projectile.
a_p	Projectile acceleration.
C_{31i}, C_{32i}, C_{33i}, C_{34i}	Functions of parameter functions defined in (22)–(25).
a_l	The ith parameter.
f	Abbreviation for $f(t)$.
$f(t)$	$\dot{B}$ coil voltage function.
f_i	Abbreviation for $f_i(t)$.
$f_i(t)$	$\dot{B}$ coil voltage function for ith segment.
$A_xF((2i-1)/2i_m)$	Current distribution segment form function.
H_e	Y distance from current element to the plane parallel to the X–Z plane containing $\dot{B}$ coil.
H_0	Y distance from X–Z plane to a plane parallel to the X–Z plane containing $\dot{B}$ coil.
$H_e(Y)$	$H_0 - Y$.
I_{g1i}, I_{g2i}, I_{g3i}, I_{g4i}	Integrals of ith segment of voltage function.
i	Subscript indicating segment number.
i_m	Number of segments.
$J(\xi, t)$	Current density in plasma.
$I(t)$	Total plasma current.
$I_i(t)$	Current in ith segment.
$L_p(t)$	Total length of plasma.
$L_i(t, \xi)$	X distance from plasma position to a plane parallel to the Y–Z plane containing $\dot{B}$ coil.
H_p	Width of the plasma armature.
$J_i(t)$	Current density of ith segment.
k	Subscript indicating data point number.
$L_{pi}(t)$	Length of ith plasma segment.
$L_{ps}(t)$	Length of plasma segment for given model.
$L_i(t, 1)$	Defined by (28).
$L_i(t, 0)$	Defined by (31).
N	Number of data points in least square fit.

Manuscript received June 29, 1987; revised December 9, 1987.
The author is with the Air Force Armament Laboratory, AFSC, Eglin AFB, FL 32542.
IEEE Log Number 8820927.

N_p	Number of parameters being fit.
l, m	Subscript indicating l, m parameter.
$P_l(t_k)$	Partial of voltage function with respect to l parameter evaluated at time t_k.
$\vec{R}_e$	Vector position of coil center in plane parallel to rails midway between rails from midpoint of current element.
$R_e(\xi, Y, t)$	Magnitude of vector $\vec{R}_e$ given in terms of ξ, Y, and t; see (4).
$R_p(\xi, t)$	$R_e(\xi, H_p, t)$.
$R_0(\xi, t)$	$R_e(\xi, 0, t)$.
$R_{pi}(t, 1)\ R_{0i}(t, 1)$	
$R_{pi}(t, 0)\ R_{0i}(t, 0)$	See (26), (27), (29), and (30).
t	Time.
t_k	kth data time point.
$\vec{W}$	Vector current element distance between rails.
W	Perpendicular distance between rails.
X, Y, Z	Coordinates; see Fig. 2.
X_i	X distance from base of ith segment.
χ_i	Variable in function; see (45).
Z_i	In function; see (43)
μ_0	Permeability of free space.
ξ	Nondimensional length of plasma segment; see (6).
γ	Plasma acceleration rate in excess of projectile; see (32)–(35).
β	Slope of current function; see (40).
V_k	Measured voltage at time t_k.
$\overline{X}$	Gaussian mean; see (42)–(45).
σ	Gaussian variance; see (42)–(45).
ϕ_e	Coil flux caused by armature current element.
$\phi(\xi, t)$	Coil flux caused by armature current sheet.

I. Introduction

THE ANALYSIS of the voltage induced on small induction ($\dot{B}$) coils has been one of the most rewarding methods of inferring the characteristics of the electromagnetic (EM) launcher armature. A closed-form mathematical model has been derived which gives the voltage induced on $\dot{B}$ coils in terms of parameters describing the current distribution in the plasma armature. The model was derived by integrating the coil magnetic field contributions (using the Biot–Savart equation for the field due to a current element) over the current elements contained in the armature model. The rate of change of the field flux linking the coil gives the coil voltage.

The parameters in the model allow the plasma to expand or contract with acceleration, to have varying total current, and to have an arbitrary functional geometrical distribution of current along the axis of the launcher. The closed-form solution requires the assumption that the current in the plasma is made up of sheets with flow being perpendicular to the rails and the current amplitude being a function of time and the distance from the projectile base. Simple predictions from the model giving the approximate position of the projectile base correlate well with optical measurements.

The important feature of the closed-form solution is that it allows the use of the method of least squares in analyzing the data. For the velocities at which we are presently operating our small launchers (1–2 km/s), a 1/2-μs data acquisition time gives about 250 meaningful data points. This is more than adequate to fit the five to nine parameters describing the model. The least square algorithm allows for the fitting or substitution of parameter measurements from other sensors. Parameters in the model obtainable from other sensors are the projectile velocity and acceleration, and the total current. Comparing the model calculations with other sensors gives a good check on the validity of the model. The model in its final form will allow the substitution of various functional forms describing the geometrical distribution of current in the axis direction. This allows the selection of the most probable function. Results from data reduction of shot data are given in a section of this paper.

The work done in this paper differs from that done in [1] in that here the equation for the voltage due to the variable current distribution was derived in closed form from the Biot–Savart equation (a fundamental equation of physics) and the algorithm was derived for fitting the model parameters using least squares.

II. Derivation of $\dot{B}$ Voltage Function

In this section the model for the voltage induced on a small $\dot{B}$ coil (oriented to pick up only the effect of the current from the armature) is derived. Fig. 1 shows the elements of the EM rail gun. Fig. 2 shows the armature, the $\dot{B}$ coil, and some of the parameters of the model. In Fig. 2 the current is flowing in the positive Z direction from the bottom to the top rail. The center of the induction coil lies midway between the rails, at a distance H_0 from the X-Z plane, and the coil plane is perpendicular to the bore axis. If we assume the coil is small and located more than about 1.5 diameters from the bore, and that the current element is perpendicular to the rails, then the Biot–Savart equation gives the field linking the coil, to a good approximation, by the following equation:

$$\vec{B}_e = (\mu_0 I_e \vec{W} \times \vec{R}_e)/4\pi R_e^3(1 + W^2/4R_e^2)$$
$$\simeq (\mu_0 I_e \vec{W} \times \vec{R}_e)/4\pi R_e^3. \quad (1)$$

The approximate form of (1) holds for a coil a few diameters from the bore and this form will be assumed to be valid in the following derivations. The flux linking the coil is

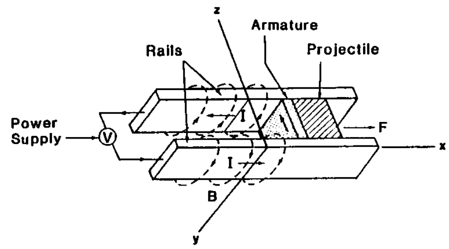


Fig. 1. Pictorial of rail gun.

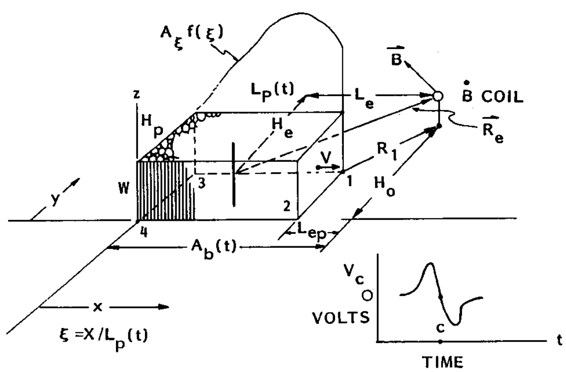


Fig. 2. Pictorial of plasma armature.

$$\phi = \vec{B}_e \cdot \vec{A}_c. \tag{2}$$

From the geometry

$$\phi_e = \mu_0 I_e W A_c H_e / 4\pi R_e^3 \tag{3}$$

where H_e is the perpendicular distance from the midpoint of the element to a plane containing the coil center and parallel to the X-Z plane. The equations may be put in a form suitable for integration by writing the parameters of the element in terms of the nondimensional variable ξ, and Y and the time t (refer to Fig. 2):

$$R_e(\xi, Y, t) = \left[\left(A_b(t) - \xi L_p(t) \right)^2 + \left(H_0 - Y \right)^2 \right]^{1/2} \tag{4}$$

$$H_e(Y) = H_0 - Y \tag{5}$$

$$\xi \equiv X(t)/L_p(t) \tag{6}$$

where $A_b(t)$ is the distance from the base of the plasma to the coil along the bore and $L_p(t)$ is the total length of the plasma. Assume ξ is independent of t.

If it is assumed that the current passing through the element is constant in a plane perpendicular to the bore axis, then

$$I_e = J(\xi, t)\, dY\, d\xi\, L_p(t). \tag{7}$$

The entire flux linking the coil may be obtained by integrating over the plasma in Y and ξ. The first integral may be obtained by substituting (4)–(7) in (3) and integrating from $Y = 0$ to $Y = H_p$. The result is

$$\phi_c(t) = C_e \int_0^1 J(\xi, t) \left[\left(1/R_p(\xi, t) \right) - \left(1/R_0(\xi, t) \right) \right] L_p(t)\, d\xi \tag{8}$$

where

$$R_p(\xi, t) \equiv R_e(\xi, H_p, t), \; R_0(\xi, t) \equiv R_e(\xi, 0, t).$$

The voltage induced on the $\dot{B}$ coil is

$$f(t) = -N \frac{d\phi_c(t)}{dt} = C_e \int_0^1 \left\{ J(\xi\ t) \left[\left(1/R_p(\xi, t) \right) - \left(1/R_0(\xi, t) \right) \right] L_p(t) \right\}' d\xi \tag{9}$$

$$C_e = -N\mu_0 W A_c / 4\pi \tag{10}$$

$$\partial[\ ]/\partial t \equiv [\ ]'. \tag{11}$$

Equation (9) is integrable in closed form if $J(\xi, t)$ is constant with respect to ξ. In order to get a closed-form integral and still simulate a variable current distribution in the bore (X) direction, the armature current is divided into segments with the current density constant over the ith segment (see Fig. 3). A one-dimensional form function will be defined to give the constant current of the ith segment in terms of the midpoint of the ith segment ($(2i - 1)/2i_m$). i_m is the number of segments. The form function is defined such that it gives the fraction of the total current carried by the ith segment and the sum of the fractional currents must equal the total current (i.e., the sum of the form function over i equals one, and the product of the form function and the total current is the ith segment current). The current density for the ith segment is given by the product of the form function for the ith segment and the total current divided by the area of the ith segment as follows:

$$J_i(t) = A_X F\left(\frac{2i - 1}{2i_m} \right) I(t) / H_p L_{pi}(t) \tag{12}$$

$$A_X \equiv 1 \Big/ \sum_{i=1}^{i_m} F\left(\frac{2i - 1}{2i_m} \right), \quad i_m = \#\text{ of segments}. \tag{13}$$

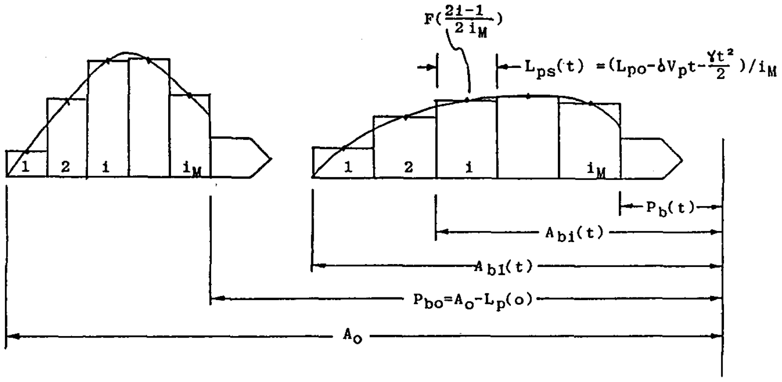
With these definitions the voltage function $f(t)$ may be approximated by integrating (9) for the ith segment and then summing over all i ($i = 1$ to i_m). Then, subscripting all variables in (9) with i, substituting from (12) into (9), performing the differentiation, extracting parameters independent of ξ from the integral, and summing over i gives the following equation:

$$\begin{aligned} f(t) = \sum_{i=1}^{i_m} C_I A_X F\left(\frac{2i - 1}{2i_m} \right) \int_0^1 & \left\{ -A_{bi}(t)\, I(t)\, L_i(t, \xi) \right. \\ & \left. + L'_{pi}(t)\, I(t)\, A_{bi}(t)\xi - L'_{pi}(t)\, I(t)\, L_{pi}(t)\xi^2 \right\} \\ & \cdot \left\{ 1/R_{pi}^3(t, \xi) - 1/R_{0i}^3(t, \xi) \right\} d\xi \\ & + I'(t)\left(1/R_{pi}(t, \xi) - 1/R_{0i}(t, \xi) \right) d\xi \end{aligned} \tag{14}$$

$$L_i(t, \xi) = A_{bi}(t) - \xi L_{pi}(t)$$

$$C_I = -N\mu_0 W A_c / 4\pi H_p \tag{15}$$

where $\xi = X_i / L_{pi}(t)$ for this integral. The integral of (14) is given below where the subscript i refers to the ith seg-



$A_{bi}(t)$ = ith PLASMA SEGMENT BASE

Fig. 3. Segmented plasma current distribution.

ment:

$$F(t) \equiv V_p(t) = \sum_{i}^{i_m} C_{li}(I_{g1i} + I_{g2i} + I_{g3i} + I_{g4i}) \equiv \sum_{i}^{i_m} f_i(t) \tag{16}$$

$$C_{li} = -N\mu_0 W A_c A_\chi F\left(\frac{2i-1}{2i_m}\right) \Big/ 4\pi H_p = C_l A_\chi F\left(\frac{2i-1}{2i_m}\right)$$

$$A_\chi = 1 \Big/ \sum_{i=1}^{i_m} F\left(\frac{2i-1}{2i_m}\right) \tag{17}$$

where the segment current is

$$I_i(t) = A_\chi F\left(\frac{2i-1}{2i_m}\right) I(t).$$

The integrals are as follows:

$$I_{g1i} = \frac{-I(t)\, A'_{bi}(t)}{L_{pi}(t)} \left\{ \left(1/R_{pi}(t,1) - 1/R_{0i}(t,1)\right) - \left(1/R_{pi}(t,0) - 1/R_{0i}(t,0)\right) \right\} \tag{18}$$

$$I_{g2i} = \frac{I(t)\, L'_{pi}(t)\, A_{bi}(t)}{L_{pi}^2(t)} \left\{ \frac{A_{bi}(t)\, L_{pi}(t) - R_{pi}^2(t,0)}{(H_0 - H_p)^2 R_{pi}(t,1)} + \frac{\left(-A_{bi}(t)\, L_{pi}(t) + R_{0i}^2(t,0)\right)}{H_0^2 R_{0i}(t,1)} + \frac{R_{pi}(t,0)}{(H_0 - H_p)^2} - \frac{R_{0i}(t,0)}{H_0^2} \right\} \tag{19}$$

$$I_{g3i} = \frac{-I(t)\, L'_{pi}(t)}{L_{pi}^2(t)} \left\{ \frac{C_{31i}}{(H_0 - H_p)^2 R_{pi}(t,1)} - \frac{C_{32i}}{H_0^2 R_{0i}(t,1)} - \ln\left(\frac{R_{0i}(t,1) + L_{pi}(t) - A_{bi}(t)}{R_{pi}(t,1) + L_{pi}(t) - A_{bi}(t)}\right) - \left(\frac{C_{33i}}{(H_0 - H_p)^2 R_{pi}(t,0)} - \frac{C_{34i}}{H_0^2 R_{0i}(t,0)} - \ln\left(\frac{R_{0i}(t,0) - A_{bi}(t)}{R_{pi}(t,0) - A_{bi}(t)}\right)\right)\right\} \tag{20}$$

$$I_{g4i} = \frac{I'(t)}{L_{pi}(t)} \left\{ \ln\left[\frac{L_i(t,1) + R_{0i}(t,1)}{L_i(t,1) + R_{pi}(t,1)}\right] - \ln\left[\frac{L_i(t,0) + R_{0i}(t,0)}{L_i(t,0) + R_{pi}(t,0)}\right] \right\} \tag{21}$$

$$C_{31i} = L_{pi}(t)\left[A_{bi}^2(t) - (H_0 - H_p)^2\right] - A_{bi}(t)\, R_{pi}^2(t,0) \tag{22}$$

$$C_{32i} = L_{pi}(t)[A_{bi}^2(t) - H_0^2] - A_{bi}(t)\, R_{0i}^2(t, 0) \tag{23}$$

$$C_{33i} = -A_{bi}(t)\, R_{pi}^2(t, 0) \tag{24}$$

$$C_{34i} = -A_{bi}(t)\, R_{0i}^2(t, 0) \tag{25}$$

$$R_{pi}(t, 1) = [L_i^2(t, 1) + (H_0 - H_p)^2]^{1/2} \tag{26}$$

$$R_{0i}(t, 1) = [L_i^2(t, 1) + H_0^2]^{1/2} \tag{27}$$

$$L_i(t, 1) = A_{bi}(t) - L_{pi}(t) \tag{28}$$

$$R_{pi}(t, 0) = [L_i^2(t, 0) + (H_0 - H_p)^2]^{1/2} \tag{29}$$

$$R_{0i}(t, 0) = [L_i^2(t, 0) + H_0^2]^{1/2} \tag{30}$$

$$L_i(t, 0) = A_{bi}(t). \tag{31}$$

If the plasma segments are defined as follows (refer to Fig. 3):

$$A_{bi}(t) = A_{b0i} - \left[(V_p + \delta V_{pi})t + \frac{(a_p + a_i)}{2} t^2\right] \tag{32}$$

where

$$A_{b0i} = A_{b0} - L_{p0}(i - 1)/i_m \tag{33}$$

$$\delta V_{pi} = (i_m - i + 1)\delta V_p / i_m \tag{34}$$

$$a_i = (i_m - i + 1)\gamma / i_m \tag{35}$$

$A_{bi}(t)$ = position of the ith plasma segment base

V_p, a_p = projectile initial velocity and acceleration

$V_p + \delta V_{pi}$, $a_p + a_i$ = velocity and acceleration of segment base

then the segment boundaries are continuous, the segments bases may have accelerations and velocities different from those of the projectile, the total length of the plasma is independent of the number of segments, and the lengths of the segments are the same independent of the segment number:

$$L_{pi}(t) = L_{ps}(t) = \left(L_{p0} - \delta V_p t - \frac{\gamma t^2}{2}\right)\Big/ i_m \quad \text{segment length} \tag{36}$$

$$L_p(t) = L_{ps}(t) i_m \quad \text{plasma length.} \tag{37}$$

With these definitions the derivatives in the voltage function are

$$A_{bi}'(t) = -[V_p + \delta V_{pi} + (a_p + a_i)t] \tag{38}$$

and

$$L_{ps}'(t) = -(\delta V_p + \gamma t)/i_m. \tag{39}$$

For the short time during the plasma passage the current may be simulated as a linear function of time:

$$I(t) = I_0 + \beta t \tag{40}$$

$$I'(t) = \beta. \tag{41}$$

The geometrical distribution of the plasma is described by the segment distribution function

$$A_\chi F\left(\frac{2i - 1}{2i_m}\right).$$

For a Gaussian function that is

$$A_\chi F(\chi_i) = A_\chi \exp(-Z_i^2/2) \tag{42}$$

$$Z_i \equiv ((\chi_i - \bar{x})/\sigma) \tag{43}$$

$$A_\chi = 1\Big/\sum_{i=1}^{i_m} \exp(-Z_i^2/2) \tag{44}$$

$$\chi_i \equiv (2i - 1)/2i_m \tag{45}$$

where $\bar{x}$ is the Gaussian mean and σ the standard deviation—the parameters of the distribution function.

It should be mentioned that the closed-form solution with five or more segments agrees with the numerical integration of (9) to four significant figures.

III. Observation on Restricted Solution

Some notion of the characteristics of the plasma may be obtained by taking the simplest solution of the voltage function and comparing this equation with actual data. For the case where there is a uniform constant current of constant length and moving at a constant velocity, there is only the first integral (in (16)) with one segment. The voltage function for these conditions is

$$f(t) = \frac{C_l I V_p}{L_p}\left\{\left(\frac{1}{R_p(t, 1)} - \frac{1}{R_0(t, 1)}\right) - \left(\frac{1}{R_p(t, 0)} - \frac{1}{R_0(t, 0)}\right)\right\}. \tag{46}$$

If the derivative of this function with respect to time is taken and the result set equal to zero, then

$$\frac{L(t, 1)}{L(t, 0)}\left[1 - \left(\frac{R_p(t, 1)}{R_0(t, 0)}\right)^{3/2}\right] - \left(\frac{R_p(t, 1)}{R_0(t, 1)}\right)^{3/2} + \left(\frac{R_p(t, 1)}{R_0(t, 0)}\right)^{3/2} = 0. \tag{47}$$

This transcendental equation is plotted in Fig. 4. The distance of the front face of the plasma (base of projectile) is plotted versus the length of the plasma. The plot shows that for a reasonable length plasma (> 1 cm) the base of the projectile is very nearly adjacent to the coil at maximum coil voltage. Experimentally, this observation is verified even though the simplest form of the voltage function was used in the derivation. One would think that much more information could be obtained about the

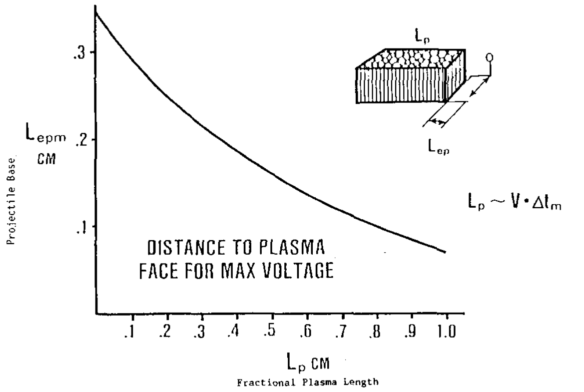


Fig. 4. Projectile base at coil maximum voltage versus length of plasma.

plasma by using the method of least squares to fit the entire voltage function for the current distribution in the plasma.

IV. Least Squares Fit to the Voltage Function

The general form for the voltage function for a data time point t_k is

$$f(t_k; a_1, a_2, \cdots, a_{np}) = \sum_{i=1}^{i_m} f_i(t_k; i; a_1, a_2, \cdots, a_{np}) \tag{48}$$

where t_k is the data point time and $a_1, a_2, \cdots, a_{np}$ are function parameters.

The least squares equation for corrections to the parameters of the function compatible with the data points is

$$\sum_{l=1}^{N_p} A_{ml} \Delta a_l = B_m, \qquad l = 1, 2, \cdots, N_p \qquad m = 1, 2, \cdots, N_p. \tag{49}$$

The solution to this set of N_p linear algebraic equations is the least squares solution for the parameter corrections, where

$$A_{lm} = \sum_{k=1}^{N} P_l(t_k)\, P_m(t_k),\; B_m = \sum_{K=1}^{N} (V_k - f(t_k)) P_m(t_k) \tag{50}$$

$$P_l(t_k) \equiv \partial f / \partial a_l$$
$$P_m \equiv \partial f / \partial a_m \tag{51}$$

and V_k = measured voltage at t_k. $f(t_k; a_1, a_2, \cdots, a_{np})$ is abbreviated by f in (51) and in some equations below.

The problem is to obtain the partial derivatives of the voltage function. The parameters which should be fitted are the initial position of the base of the plasma A_{b0}, the initial length of the plasma L_{p0}, the expansion rate of the plasma δV_p, the velocity of the projectile V_p, and the parameters describing the geometrical distribution such as $\bar{x}$ and σ for the Gaussian. Other parameters which may require fitting are the acceleration of the projectile a_p and the accelerated expansion of the plasma γ. Parameters such as the area of the coil A_c may be fitted to check on the validity of the model (the projectile velocity may also be measured by other sensors and also be used as a check).

The area of the coil appears explicitly and only in the voltage function. The parameters of the form function $(\bar{X}, \sigma)$ appear only in this function. The remainder of the parameters appear explicitly only in the parameter functions, (32)–(41). The parameter functions appear in the primary voltage functions, (18)–(21), and in some cases in the auxiliary functions, (22)–(31). So in general the parameter partials must be obtained (other than the coil diameter and geometrical form parameters) by taking the partial derivative of the voltage function with respect to the parameter functions and the auxiliary functions and working down to the parameters. For example, the partial of the voltage function with respect to the projectile velocity has 16 terms, the first being

$$\begin{aligned}
\frac{\partial f \partial A_{bi}}{\partial A_{bi} \partial V_p} &= \sum_{i=1}^{i_m} C_{li} \frac{\partial f_i}{\partial A_{bi}} \frac{\partial A_{bi}}{\partial V_p} \\
&= \sum_{i=l}^{i_m} C_{li} \Bigg\{ \frac{I(t_k)\, L'_{pi}(t_k)}{L_{pi}^2(t_k)} \left(\frac{A_{bi}(t_k)\, L_{pi}(t_k) - R_{pi}^2(t_k, 0)}{(H_0 - H_p)^2 R_{pi}(t_k, 1)} + \frac{-A_{bi}(t_k)\, L_{pi}(t_k) + R_{0i}^2(t_k, 0)}{H_0^2 R_{0i}(t_k, 1)} \right. \\
&\quad \left. + \frac{R_{pi}(t_k, 0)}{(H_0 - H_p)^2} - \frac{R_{0i}(t_k, 0)}{H_0^2} \right) + \frac{I(t_k)\, L'_{pi}(t_k)\, A_{bi}(t_k)}{L_{pi}(t_k)} \left(\frac{1}{(H_0 - H_p)^2 R_{pi}(t_k, 1)} \right. \\
&\quad \left. - \frac{1}{H_0^2 R_{0i}(t_k, 1)} \right) - \frac{I(t_k)\, L'_{pi}(t_k)}{L_{pi}^2(t_k)} \left[- \frac{1}{R_{pi}(t_k, 1) + L_{pi}(t_k) - A_{bi}(t_k)} \right. \\
&\quad \left. + \frac{1}{R_{0i}(t_k, 1) + L_{pi}(t_k) - A_{bi}(t_k)} + \frac{1}{R_{pi}(t_k, 0) - A_{bi}(t_k)} - \frac{1}{R_{0i}(t_k, 0) - A_{bi}(t_k)} \right] \Bigg\} (-t_k).
\end{aligned} \tag{52}$$

Obviously, the partials are very complicated. However, after a few are worked out, many of the others are repetitious. The partials which have been worked out, pro-

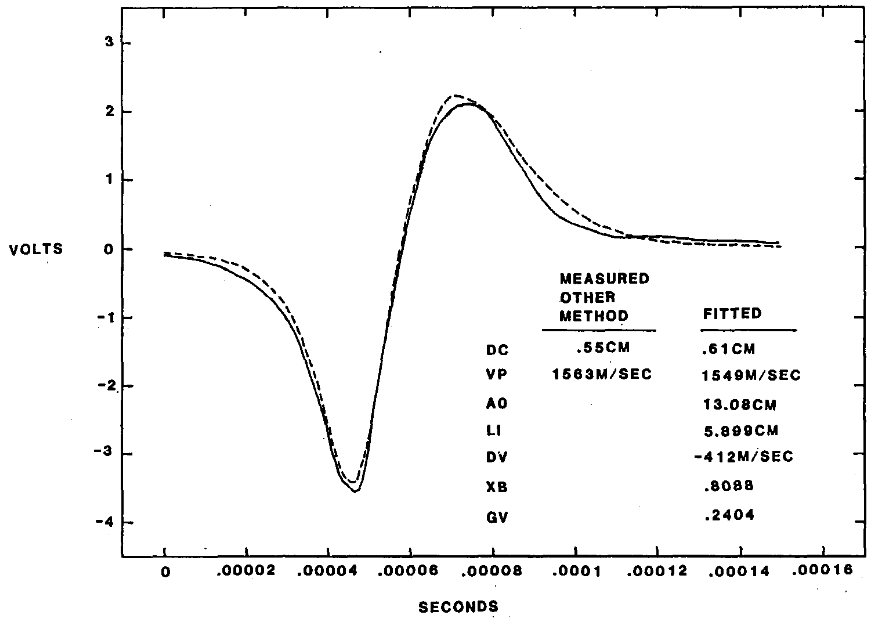


Fig. 5. Data fit of $\dot{B}$ voltage versus time.

grammed, and debugged are most of those mentioned above (i.e., A_c, A_{b0}, L_{p0}, V_p, δV_p, and the Gaussian function parameters $\overline{X}$, σ). The partials for the Gaussian parameters are rather simple and are as follows:

$$\frac{\partial f}{\partial \overline{X}} = \sum_i^{i_m} f_i \frac{1}{\sigma} \left\{ Z_i - A_X \sum_{i=1}^{i_m} Z_i \exp\left(-Z_i^2/2\right) \right\}$$

$$\frac{\partial f}{\partial \sigma} = \sum_i^{i_m} f_i \frac{1}{\sigma} \left\{ Z_i^2 - A_X \sum_{i=1}^{i_m} Z_i^2 \exp\left(-Z_i^2/2\right) \right\}. \quad (53)$$

The least squares fit has been programmed on a Zenith 248 (PC compatible) microprocessor and compiled in basic runs one iteration on seven parameters using five segments for 300 data points in 1.5 h. A program is being completed now which will run about 20 times faster.

Before discussing the fitting of actual test data, it should be mentioned that the derived least squares fitting algorithm was adequately tested on generated data. Data were generated using the closed-form solution and selected values for the parameters. The parameters were then changed and the leasts square algorithm was used to correct the incorrect parameters using the generated data. The algorithm corrected the parameter values back to the generated data values to the accuracy of the microcomputer within a few iterations. When the original data were perturbed with white noise the algorithm brought the parameters back to the generated data values consistent with the amplitude of the white noise.

IV. Fitting Test Data

In the following discussion the $\dot{B}$ data fitted were obtained from the firing of a 1-cm, square bore EM rail gun. The parameters to be fitted are the area of coil (which can be converted to a diameter), the projectile velocity V_p, the initial plasma base position A_{b0}, the initial length L_{p0}, the expansion rate δV_p of the plasma, and the parameters of the Gaussian current distribution $\overline{X}$ and σ.

The $\dot{B}$ coil was located 35.5 cm from the base of the square bore 1-cm rail gun. The coil diameter was 0.55 cm, and its center was located 2.2 cm from the bore center with its plane oriented to pick up the armature current only (all dimensions are approximate).

Four flash X-ray heads were used 1-m downrange from the muzzle to obtain exit velocity. A Nicolet oscilloscope was used to obtain voltage data from the $\dot{B}$ coil at an acquisition period of 0.5 μs. As the plasma armature passes the $\dot{B}$ coil, the characteristic signal (Fig. 2) is obtained. Voltages above 50 mV were used which gave 300 data points.

The plotted raw data and the fitted data are shown in Fig. 5. Also shown in Fig. 5 is a table giving the fitted values for the seven parameters and the value of the diameter of the coil D_c and projectile velocity V_p measured by other methods. The coil measurement is a ruled measurement and not an effective calculated value. The velocity measurement is taken from the peak voltage on two $\dot{B}$ coils.[1] As is shown, the measured values and fitted parameters compare very well. The parameters giving the current distribution are the two Gaussian parameters ($\overline{X}$ and σ) and the plasma spread velocity δV_p. A plot of the segment current distribution is given in Fig. 6.

V. Conclusions

A closed-form solution for the voltage induced on small $\dot{B}$ coils due to the current in a plasma rail gun was derived using the Biot–Savart equation. The method of least

[1]The peak voltage on the $\dot{B}$ coil occurs 6 μs after the initial rise on the apposing photodiode which indicates that the approximate theory on the correlation between projectile base passage and $\dot{B}$ maximum voltage is very good.

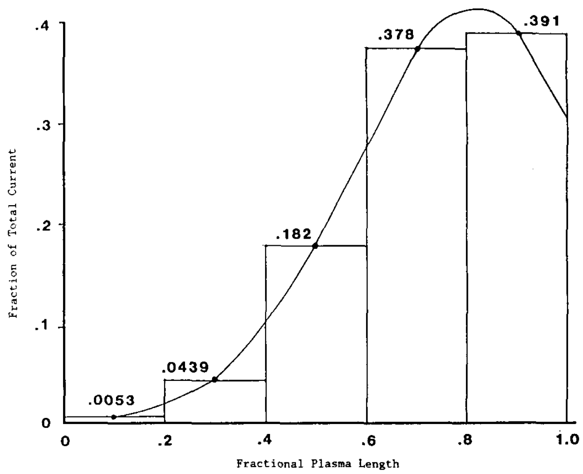


Fig. 6. Fraction current segment amplitude versus fractional plasma length.

squares was applied to this solution to obtain the parameters describing the current distribution in the plasma arc. This was accomplished by computer programming the derived algorithm. Seven parameters were fit to an actual data set containing 300 points. Two of the fitted parameters, the diameter of the coil and the projectile velocity, were compared to measurements taken by other means. The comparisons were good.

The fitted parameters which describe the current distribution (δV_p, $\overline{X}$, and σ) were obtained. There is no way to check these parameters with other sensors. The values obtained appear reasonable. The $\overline{X}$ and σ give a geometrical current distribution within the bounds of the one-dimensional equilibrium model [2].

The $\dot{B}$ voltage model is a useful tool which allows rapid computer analysis in comparing plasma current characteristics for different conditions immediately after a shot. The current distribution parameters appear to give reasonable approximations to the plasma current distribution.

References

[1] K. A. Jamison, M. Marquez-Reines, and H. S. Burden, "Measurements of the spatial distributions of current in a rail gun arc armature," *IEEE Trans. Magn.*, vol. MAG-20, no. 2, pp. 403-406, Mar. 1984.

[2] J. D. Powell and J. H. Batteh, "Plasma dynamics of an arc-driven electromagnetic, projectile accelerator," *J. Appl. Phys.*, vol. 52, no. 4, pp. 2717-2730, Apr. 1981.