

Analysis of Inductive Coil Gun Performance Based on Field Coupling Circuit Method

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Abstract-Inductive coil gun's performance analysis is very important for experimental research and its electromagnetic optimization design. Circuit simulation is carried out based on current filament model, the governing equations of current filament model may boil down to initial value problem of nonlinear variable coefficient ordinary differential equations, which can be solved by numerical method and is easy to program. From circuit simulation, the dependence of system performance on variable parameter can be found. Field model is built based on composite grid method; composite grid method uses two sets of mutual independent grids to discrete solution region, it overcomes the trouble of remeshing in normal finite element technique based on one set of grid. In this paper performance analysis of three stages coaxial inductive coil gun is carried out based on field coupling circuit method. Three dimensional moving conductor eddy current field simulation of coil gun is realized, field variables distribution in the launching process is obtained. By comparing the results of circuit model and field model the validity of the two models is proved.

I. INTRODUCTION

Coil launcher is a kind of specific induction motor [1]. Generally, it constitutes by five parts: pulse power, high-speed switch, exciting coil, armature and projectile. The pulse power loads electromagnetic energy on exciting coil after pulsed on. Transient magnetic field produced by the exciting coils induces circumferential eddy current in the armature. The circumferential eddy current interacts with radial component of the magnetic field which produces Lorentz force to drive the armature and projectile in the direction of muzzle. When armature arrives at an appropriate position, the next stage is fired, this process continues until the projectile is out of bore.

Although working principle of coil gun is simple, coil gun's launching is a complex electromagnetic transient process, to obtain main performance parameters and realize optimization design is not easy. Compared with expensive prototype experiments, numerical simulation can save design cost and improve work efficiency greatly in the optimized design of coil gun. The numerical simulation model for coil gun can be divided into two types: circuit model and multi-physical field model [2]. The circuit model has the virtue of simple and easy realization. According to the circuit model, dependence of variable parameters on result can be found easily. The multi-physical field model solves the problem from partial differential equation or integral equation, and can obtain various

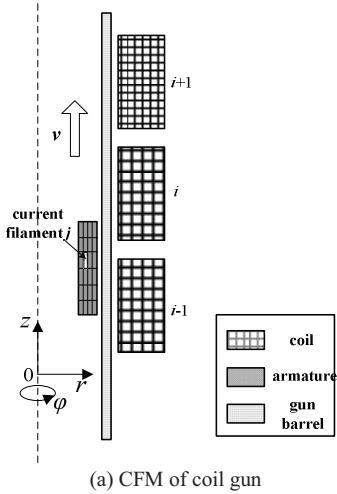
field quantities accurately, such as magnetic flux density and eddy current density.

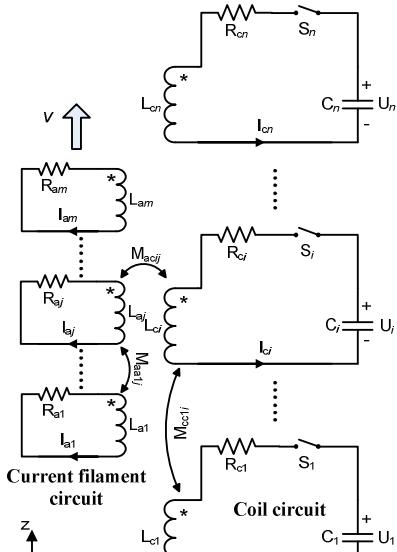
In the analysis of coil gun only use circuit model can't get the field distribution, so precise analysis is impossible; only use field model can't complete the whole analysis procedure, since the current of exciting coils which is material property in field model doesn't know in advance. So it is absolutely essential to analyze coil gun based on field and circuit combined model.

II. BASIC PRINCIPLES OF CIRCUIT MODEL AND FIELD MODEL

A. Current filament model

There are two types of circuit model: current sheet model [3] and current filament model (CFM) [4]-[5]. The current sheet model replaces the currents in an actual barrel and in a cylindrical tubular projectile (sleeve) with two equivalent current sheets located at their equivalent radius; by computing the magnetic field produced by the two current sheets the Lorentz force applied on projectile can be obtained. The current filament model divides massive conductive parts of the system into elementary volume elements, in which uniform distribution of current is assumed [6], a current filament is associated with every volume element, and its electrical parameters are calculated, the electrical and mechanical equations governing the behaviors of the system are formulated on the basis of the adopted equivalent network (the schematic diagram of CFM is shown in Fig.1). Considering the definite physical meaning and simple solving process of current filament model, this paper uses it to establish circuit model.





(b) Equivalent circuit diagram of CFM
Fig. 1. Schematic diagram of CFM

As shown in Fig.1(b), there are n stages of existing coils and the armature was divided into m current filaments. According to kirchhoff's voltage law, we can write out the governing equations as

$$[V] = [R][I] + \frac{d}{dt} \{ [L][I] + [M][I] \} \quad (1)$$

$[V]$ and $[I]$ are column vectors which have $m+n$ elements and denote the voltage vector and current vector, respectively. $[R]$ and $[L]$ are $m+n$ order diagonal matrixes composed of the individual projectile and coil resistances and self-inductances, respectively, and $[M]$ is a square $(m+n) \times (m+n)$ matrix, each element of which represents a mutual inductance between any two individual coils. It should be noted that the mutual inductances $[M]$ between drive coils and projectile coils are functions of the distance which denotes the position of the armature and is a function of time.

Capacitors voltage and coils current satisfy the relations

$$[C] \frac{d}{dt} [V_c] = -[I_d] \quad (2)$$

where $[C]$ is a n -order diagonal matrix containing the capacitances. $[V_c]$ and $[I_d]$ are the capacitor voltages and the exciting coils currents column matrices respectively. $[V_c]$ and $[I_d]$ are submatrices of $[V]$ and $[I]$.

The Lorentz force acting on the projectile is given by

$$F = \frac{1}{2} [I]^T [G] [I] \quad (3)$$

where $[G] = d[M]/dx$ and the superscript T stands for the transpose of the matrix. Finally, combine the equations of motion with (1)-(3). The complete set of government equations which describe the capacitively driven inductive coil gun system is given in matrix form:

$$\{[L] + [M]\} \frac{d}{dt} [I] = [V] - [R][I] - v_p [G][I] \quad (4)$$

$$[C] \frac{d}{dt} [V_c] = -[I_d] \quad (5)$$

$$M_p \frac{dv_p}{dt} = \sum_{p=1}^m \sum_{d=1}^n I_p I_d \frac{dM_{pd}}{dx} \quad (6)$$

$$\frac{dx}{dt} = v_p \quad (7)$$

where M_p is the mass of armature and v_p is the velocity of armature. So the governing equations represent a set of simultaneous nonlinear differential equations with time-variable coefficients which can be solved by fourth-order Runge-Kutta method. The number of unknown variables in the governing equations is

$$N = 2n + m + 2 \quad (8)$$

m filament currents, n exciting coil currents, n capacitor voltages, v_p , and x

B. Composite Grid Method

Composite grid method (CGM) developed from the conventional multi-grid method uses two grid sets: one coarse grid for the global region (Ω_c) and one fine grid for the local region (Ω_f), $\Gamma = \Omega_c \cap \Omega_f$ is the interface between the two regions[7], as shown in Fig. 2.

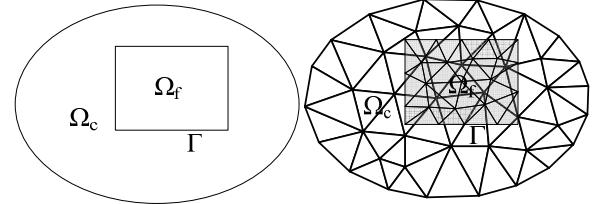


Fig. 2. Solution region and two sets of grids of CGM

The governing equations are written in operator forms below

$$\begin{cases} L_c v = f_c & \Omega_c \\ L_f u = f_f & \Omega_f \\ v|_{\Gamma} = u|_{\Gamma} \end{cases} \quad (9)$$

Where w and u are unknowns, L_c and L_f denote the operators, f_c and f_f denote the source items in each region, respectively.

According to the principle of virtual work, (9) can be rewritten as

$$(L_c v, \bar{v})_{\Omega_c} = (f_c, \bar{v})_{\Omega_c} \quad (10)$$

$$(L_f u, \bar{u})_{\Omega_f} = (f_f, \bar{u})_{\Omega_f} \quad (11)$$

By (10) and (11), it is obvious to obtain the following equation

$$(L_c v - f_c, \bar{v})_{\Omega} = (L_c v - f_c, \bar{v})_{\Omega_f} - (L_f u - f_f, \bar{u})_{\Omega_f} \quad (12)$$

The computation scheme in CGM can be written as

$$(L_c v - f_c, \bar{v})_{\Omega} = (L_c u_c - f_c, \bar{v})_{\Omega_f} - (L_f u_f - f_f, \bar{u})_{\Omega_f} \quad (13)$$

where u_c and u_f denote the variable in Ω_f associated with the operators L_c and L_f respectively.

The basis functions in the global and local grids are

$$\Psi = \{\Psi_j\}_{j=1}^{n_c} \text{ and } \Phi = \{\Phi_j\}_{j=1}^{n_f}, \text{ where } n_c \text{ and } n_f \text{ are the}$$

numbers of nodes in each set of grids. Variable u , v , \bar{u} and \bar{v} can be expressed as

$$v = \sum_{j=1}^{n_c} v_j \Psi_j = \mathbf{V}^T \Psi, \quad \bar{v} = \sum_{j=1}^{n_c} \bar{v}_j \Psi_j = \bar{\mathbf{V}}^T \Psi \quad (14)$$

$$u = \sum_{j=1}^{n_f} u_j \Phi_j = \mathbf{U}^T \Phi, \quad \bar{u} = \sum_{j=1}^{n_f} \bar{u}_j \Phi_j = \bar{\mathbf{U}}^T \Phi \quad (15)$$

Define a matrix \mathbf{D} as

$$\Psi = \mathbf{D} \Phi \quad (16)$$

Equation (16) develops the mapping relation between the global and local grids. It is satisfied in Ω_f that

$$v = \mathbf{V}^T \mathbf{D} \Phi \quad (17)$$

Equation (13) can be expanded as

$$\int_{\Omega} \Psi L_c(\Psi^T) d\Omega \mathbf{V} = \int_{\Omega} f_c \Psi d\Omega + \mathbf{D}(\mathbf{R}_c - \mathbf{R}_f) \quad (18)$$

$$\mathbf{R}_c = \int_{\Omega_f} \Phi L_c(\Phi^T) d\Omega \mathbf{U}_c - \int_{\Omega_f} \Phi f_c d\Omega \quad (19)$$

$$\mathbf{R}_f = \int_{\Omega_f} \Phi L_f(\Phi^T) d\Omega \mathbf{U}_f - \int_{\Omega_f} \Phi f_f d\Omega \quad (20)$$

The correction term $\mathbf{R}_c - \mathbf{R}_f$ is calculated in the local grid. The addition of this local grid term to the equation of global grid is realized by multiplying a \mathbf{D} matrix in front of it

The nodal result \mathbf{U}_c and \mathbf{U}_f are the solutions of two Dirichlet boundary value problems in Ω_f .

$$(L_c u_c, \bar{u})_{\Omega_f} = (f_c, \bar{u})_{\Omega_f} \quad (21)$$

$$(L_f u_f, \bar{u})_{\Omega_f} = (f_f, \bar{u})_{\Omega_f} \quad (22)$$

It should be pointed out that the Dirichlet boundary condition is imposed to the matrix equation in finite element procedure. The values of \mathbf{R}_c and \mathbf{R}_f are non-zero at the degrees of freedom belonging to the Dirichlet boundary.

The Dirichlet boundary conditions of (21) and (22) are interpolated through the results calculated in the global grid. continuity boundary condition in (9) is weakly satisfied by

$$(u - v, \bar{u})_{\Gamma} = 0 \quad (23)$$

which can be written as

$$\bar{\mathbf{U}}^T \int_{\Gamma} \Phi \Phi^T d\Gamma \mathbf{U} = \bar{\mathbf{U}}^T \int_{\Gamma} \Phi \Phi^T d\Gamma \mathbf{D}^T \mathbf{V} \quad (24)$$

So the mapping from global grid to local grid is through the relation

$$\mathbf{U} = \mathbf{D}^T \mathbf{V} \quad (25)$$

The calculation procedure of CGM is shown in Fig. 3, the iteration continues until the results' convergence is achieved in every time-step. Since the two grid sets are created independently, CGM overcomes the trouble caused by only one set of grid and it is very convenient to handle with MCEC

problems [8].

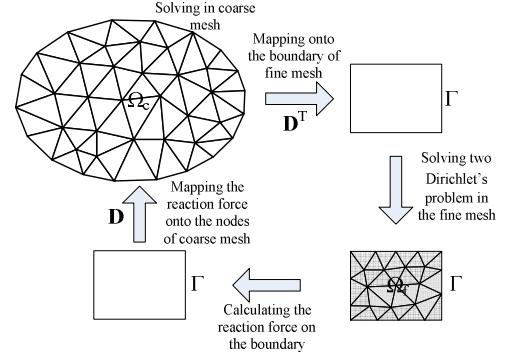


Fig. 3. Calculation procedure of CGM

III. COUPLE THE TWO METHOD IN ANALYSIS OF COIL GUN

In order to couple the two methods in the analysis of coil gun, a three-stage coaxial inductive coil gun simulation model is built. The parameters of exciting coils and armature are listed in table I.

TABLE I
PARAMETERS OF EXCITING COILS AND ARMATURE

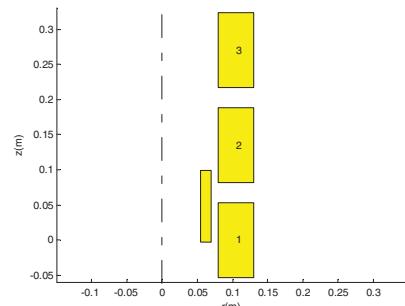
Coil	Inside radius	80mm
	Radial thickness	50mm
	Axial length	107mm
	Spacing distance	28mm
	Turn number of wire	38
Armature	Inside radius	55mm
	Radial thickness	15mm
	Axial length	103mm
	Mass	1kg
	Conductivity	3.54×10^7 S/m
	Differential permeability	1
	Initial velocity	0m/s

The pulse capacitors' parameters are listed in table II.

TABLE
PARAMETERS OF CAPACITOR

Capacitor number	Capacitance (mF)	Charge voltage (kV)
1	20.485	4.6
2	5.863	9.1
3	2.059	14.7

At first we use circuit model to calculate some parameters in the launching process, such as currents of each coil, force acting on the armature, velocity of armature. The structure diagram of coil gun and division of current filament is shown in Fig. 4. Current waveform of exciting coils and current filaments obtained from circuit simulation are shown in Fig. 5.



(a) Structure diagram of three-stage coil gun

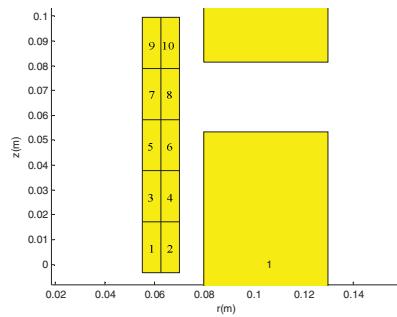


Fig. 4. Structure diagram of coil gun and division of current filament

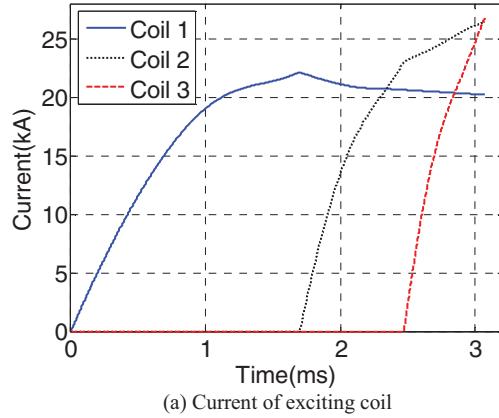
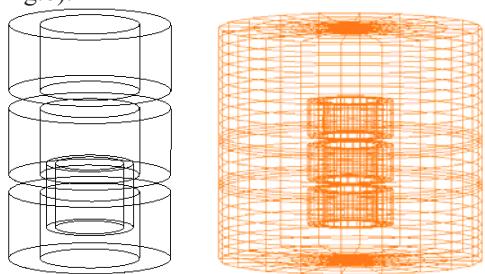
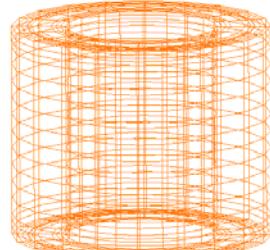


Fig. 5. Current waveform of circuit model

Then the 3D field model based on CGM is built as shown in Fig. 6. The currents got by circuit model are loaded in field model as material properties of exciting coils. By solving the field model field quantities such as eddy current field, magnetic flux density at different time can be obtained (as displayed in Fig. 7 and Fig. 8).



(a) 3D views (b) Global region meshed in coarse grids



(c) Armature meshed in fine grids
Fig. 6. Field model and domain mesh

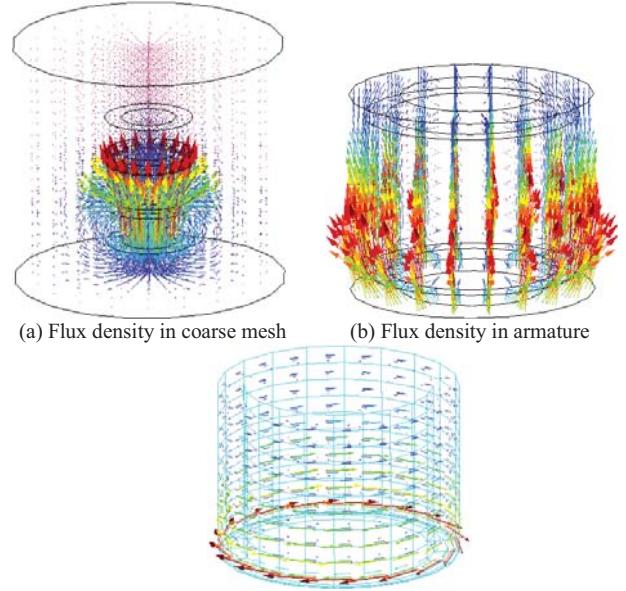


Fig. 7. Field quantities distribution at 0.60ms

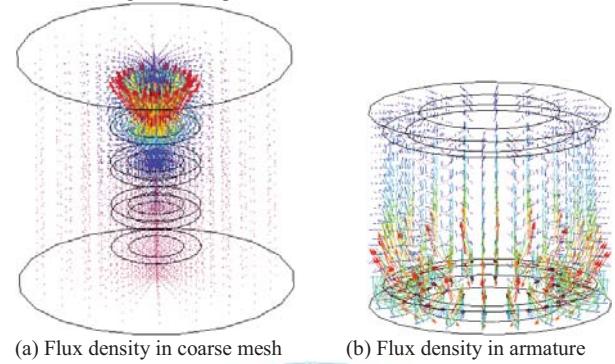
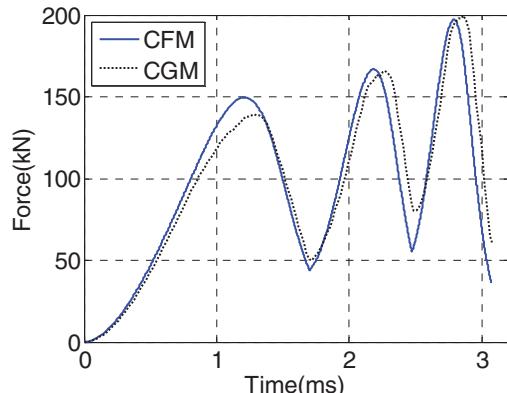


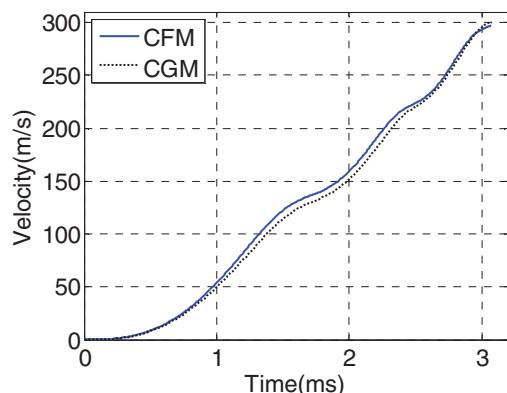
Fig. 8. Field quantities distribution at 2.96ms

From Fig. 7 and Fig. 8, we can see that traveling magnetic field produced by exciting coils moves ahead with armature. The induced eddy current mainly distributes in the back end of the armature which shows agreement with circuit model (as

shown in Fig.5 (b)). Results of the two models are compared (as shown in Fig. 9) and the validity of the coupling method is proved.



(a) Force curve of CFM and CGM



(b) Velocity result of CFM and CGM

Fig. 9. Results comparison of circuit model and field model

IV. CONCLUSION

This paper described a kind of specific induction motor that is inductive coil gun, introduced its circuit model based on current filament method and field model based on composite grid method, built a three-stage coaxial inductive coil gun and simulated it according to “field-circuit” combined technique. By comparing the results obtained by the two methods the validity of field combined circuit method is proved. The method presented in this paper is very important and useful for precise design of inductive coil gun.

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