Capacitor-Driven Coil-Gun Scaling Relationships

Zhang Yadong, Wang Ying, and Ruan Jiangjun

***Abstract*—Scaling method is an applied scientific method employed in experimental analysis. Also, in the analysis of an electromagnetic-launch system, the scaling method can be an aid in the conversion of test results from models to original. This paper focuses on the capacitor-driven coil-gun scaling relationships between the original and the model. The current-filament method is used to derive the scaling relationships of a capacitor-driven coil gun. If the input scale factors are chosen properly, output scale factors can be calculated by the scaling-relationship equations. Two single-stage coil guns were constructed and tested to verify the scaling method. The calculated scale factors agreed quite well with the test data. Further, three-stage coil guns were simulated based on Ansoft to verify the scaling method of the field results and changes in velocity with respect to the discharge position. It is shown that the scaling method is feasible. These laws allow, with proper application, the construction of experimental models in which phenomena similar to those occurring in the original are reproduced. The result can then be recalculated, using the scale factor for the physical quantity, into the original configuration. A detailed derivation and validation will be presented in this paper.**

***Index Terms*—Coil gun, current-filament model, model, scaling.**

I. Introduction

A COAXIAL capacitor-driven coil gun consists of a barrel formed by an array of stationary coils, which create a magnetic field for propelling an armature. The coils of the barrel are fed in sequence by a set of capacitor-driven circuits. Compared with a rail gun, a coil gun has virtues of high efficiency, no sliding electrical contact, and suitability for launching massive objects [1]–[4].

The scaling method has been widely used in electromagnetic-launch technology [5]–[7]. Due to resource constraints, conducting experiments in models makes economical sense. Model-system test results and the experiences gained during the fabrication and testing can be extremely valuable. However, how to extrapolate the result from model tests to original constructions is more essential. As a bridge, the scaling method can set up the relationships between the model and the original. Once scaling relationships are established, the performance of the original construction can be reproduced by experiment on the scaled model. The result can be recalculated, using the scale factor for the physical quantity, into the original configuration. This paper, based on the current-filament method and some electromagnetic-field equations, deduced the scaling relationships of a capacitor-driven coil gun. Then, two single-stage coil guns were constructed and tested to verify the scaling method. Further, finite-element simulations based on Ansoft were constructed to verify the scaling relationships between the two models, including circuit and field results.

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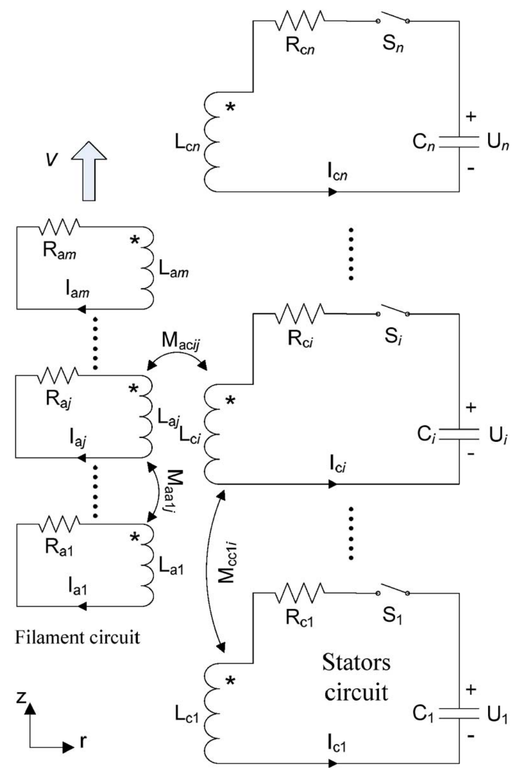


Fig. 1. Filament model of the capacitor-driven coil gun.

II. Derivation of the Coil-Gun Scaling Relationships

In a coil gun, forces are characterized by the current and the variations in the mutual inductance between the armature and coils. The mutual inductance could be determined using the current-filament method described in [8]. It assumes an axisymmetric accelerator geometry. The set of equations can be derived by considering the induction acceleration of a circular filament, driven in a repulsion mode by another concentric circular filament subjected to an arbitrarily varying voltage. An induction coil gun based on the current-filament method is modeled by the equivalent circuit shown in Fig. 1.

Employing Kirchhoff law, the network equations of the ith stator coil is expressed as

$$R_{ci}I_{ci} + L_{ci}\frac{dI_{ci}}{dt} + \sum_{k=1}^{i-1} M_{cck i}\frac{dI_{ck}}{dt} + \sum_{j=1}^{m} \frac{d}{dt}(M_{caij}I_{aj}) = U_i \tag{1}$$

where

$$U_i = U_{i0} - \frac{1}{C_i}\int_{t_i}^{t} I_{ci}\, dt, \qquad t \geq t_i. \tag{2}$$

Employing Kirchhoff law, the network equations of the jth filament is expressed as

$$R_{aj}I_{aj} + L_{aj}\frac{dI_{aj}}{dt} + \sum_{k=1}^{i}\frac{d}{dt}(M_{ackj}I_{ck}) + \sum_{k=1,k\neq j}^{m} M_{aajk}\frac{dI_{ak}}{dt} = 0. \tag{3}$$

The basic coil-gun equations are expressed as

$$F = \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ci}I_{aj}\frac{dM_{acij}}{dz} = ma \tag{4}$$

$$v = \int a\, dt \tag{5}$$

$$l = \iint a\, dt\, dt. \tag{6}$$

The resistance equation is expressed as

$$R = \rho l_0 / s. \tag{7}$$

The self-inductance and mutual inductance are

$$L \propto f(\mu, x), \quad M \propto f'(\mu, x). \tag{8}$$

The efficiency of a coil gun is calculated as

$$\eta = \frac{\frac{1}{2}m_0 v^2}{\frac{1}{2}\sum_{i=1}^{n} C_i U_0^2} = \frac{m_0 v^2}{\sum_{i=1}^{n} C_i U_0^2}. \tag{9}$$

The electromagnetic-field equations for the system can be written as

$$\nabla \times \frac{B}{\mu} = J \tag{10}$$

$$\nabla \times \frac{J}{\sigma} = -\frac{\partial B}{\partial t} \tag{11}$$

$$\nabla \cdot B = 0 \tag{12}$$

where

- U_i voltage applied to the ith coil;
- R_{ci} resistance of the ith coil;
- R_{aj} resistance of the jth armature;
- L_{ci} self-inductance of the ith coil;
- L_{aj} self-inductance of the jth armature;
- M_{ccki} mutual inductance between the ith and kth coil;
- M_{aajk} mutual inductance between the jth and kth armature;
- M_{caij} mutual inductance between the ith coil and jth filament;
- I_{ci} current in the ith coil;
- I_{ck} current in the kth coil;
- I_{aj} current in the jth filament;
- I_{ak} current in the kth filament;
- B magnetic flux density,
- J current density,
- t time,
- σ electrical conductivity,
- μ permeability.

As a prerequisite, the model must be subjected to physical processes similar to those existing in the original. The equation systems for the original and the model are the same and differ only in the indexes of the physical quantities. We denote as () and (a) the coordinates associated with the original and model, respectively. Scale factor K used to describe the scaling relationship is defined as $()' = K\,()$, i.e., $B' = K_B\, B$, etc. All homologous values of the original and the model must, for each quantity, maintain a constant proportionality to each other. With this notation, the equations for the model can be written as

$$K_{R_{ci}}K_{I_{ci}}R_{ci}I_{ci} + K_{M_{ccki}}\frac{K_{I_{ck}}}{K_t}\sum_{k=1}^{i-1}M_{ccki}\frac{dI_{ck}}{dt} + \cdots$$
$$K_{L_{ci}}\frac{K_{I_{ci}}}{K_t}L_{ci}\frac{dI_{ci}}{dt} + K_{M_{caij}}\frac{K_{I_{aj}}}{K_t}\sum_{j=1}^{m}\frac{d}{dt}(M_{caij}I_{aj}) = K_{U_i}U_i \tag{1a}$$

$$K_{U_i}U_i = K_{U_{i0}}U_{i0} - \frac{K_{I_{ci}}K_t}{K_{C_i}}\frac{1}{C_i}\int_{t_i}^{t} I_{ci}\, dt \tag{2a}$$

$$K_{R_{aj}}K_{I_{aj}}R_{aj}I_{aj} + K_{M_{ackj}}\frac{K_{I_{ck}}}{K_t}\sum_{k=1}^{i}\frac{d}{dt}(M_{ackj}I_{ck}) + \cdots$$
$$K_{L_{aj}}\frac{K_{I_{aj}}}{K_t}L_{aj}\frac{dI_{aj}}{dt} + K_{M_{aajk}}\frac{K_{I_{ak}}}{K_t}\sum_{k=1,k\neq j}^{m}M_{aajk}\frac{dI_{ak}}{dt} = 0 \tag{3a}$$

$$K_F F = \frac{K_{I_{ci}}K_{I_{aj}}K_{M_{acij}}}{K_l}\sum_{i=1}^{n}\sum_{j=1}^{m}I_{ci}I_{aj}\frac{dM_{acij}}{dl} = K_m K_a ma \tag{4a}$$

$$K_v v = K_a K_t \int a\, dt \tag{5a}$$

$$K_l l = K_a K_t^2 \iint a\, dt\, dt \tag{6a}$$

$$K_R R = \frac{K_\rho K_{l_0}}{K_s}\frac{\rho l_0}{s} \tag{7a}$$

$$K_L' L \propto K_\mu K_x f(\mu, x), \quad K_M M \propto K_\mu K_x f'(\mu, x) \tag{8a}$$

$$K_\eta \eta = \frac{K_m K_v^2}{K_{C_i}K_{U_0}^2}\frac{mv^2}{\sum_{i=1}^{n} C_i U_0^2} \tag{9a}$$

$$\frac{K_B}{K_x K_\mu}\nabla \times \frac{B}{\mu} = K_J J \tag{10a}$$

$$\frac{K_J}{K_x K_\sigma}\nabla \times \frac{J}{\sigma'} = -\frac{K_B}{K_t}\frac{\partial B}{\partial t} \tag{11a}$$

$$\frac{K_B}{K_x}\nabla B = 0. \tag{12a}$$

Physical similarity entails the condition that the equations for the original and the model differ only by a constant. This means that the power products of the scale factors must equal

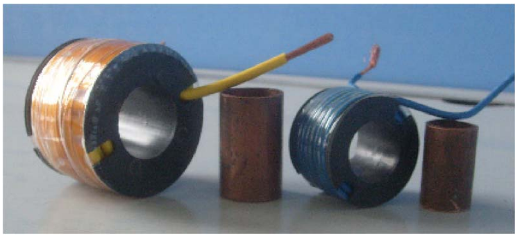


Fig. 2. Original model and scale model of the experiment.

each summand in (1a)–(12a). For example, the conditions on the scale factors of (1a) can be expressed with the following:

$$K_{R_{ci}} K_{I_{ci}} = K_{M_{ccki}} \frac{K_{I_{ck}}}{K_t} = K_{L_{ci}} \frac{K_{I_{ci}}}{K_t} = K_{M_{caij}} \frac{K_{I_{aj}}}{K_t} = K_{U_i}. \tag{13}$$

With this notation, one could obtain the condition equations for the scale factors from (1a)–(12a). Because a coil-gun system is often constructed by linear materials, the permeability and electrical conductivity of the model and the original are the same. We could set $K_\rho = K_\sigma = K_\mu = 1$ as the preconditions. Finally, the scaling relationships can be transformed and summarized as (14).

The solutions of these equations give the scale laws which govern the relationships of the scale factors with each other for the physical quantities. If a coil-gun model is constructed to research the original, the scaling relationships of (14) must be followed, where K_t must be equal to the square of the geometric scale factor K_x, and the gun current-peek scale factor K_I must be equal to $K_{U0} K_x$, etc. The original and the model have the same efficiency. The scale factors could be selected flexibly based on different experimental conditions. The results gained from experiments on the model can now be converted into the original

$$\begin{cases} K_\eta = K_\rho = K_\mu = K_\sigma = 1 \\ K_v = K_R = K_{R_{ci}} = K_{R_{aj}} = 1/K_x \\ K_M = K_L = K_r = K_{l1} = K_{l2} = K_{r_0} = K_z = K_x \\ K_t = K_x^2 \\ K_{C_i} = K_x^3 \\ K_B = K_{U_i} = K_{U_0} \\ K_{I_{ck}} = K_{I_{aj}} = K_{I_{ak}} = K_{I_{ci}} = K_{U_0} K_x \\ K_J = K_{U_0}/K_x \\ K_{m_0} = K_{U_0}^2 K_x^5. \end{cases} \tag{14}$$

III. Experimental Validation

To verify the scaling relationships of a coil gun, two single-stage coil guns were constructed, as shown in Fig. 2. The geometry parameters of the two models are shown in Fig. 3. The geometry scale factor is $K_x \approx 0.8$. The mass of the original armature is 40 g, and the scaled armature is 20 g. Therefore, the mass scale factor is $K_m = 0.5$. According to (14), we could get $K_C \approx 0.5$ and $K_{U0} \approx 1.24$. The power source that we used and the results that we tested are shown in Table I. The scale factors of current peek, time, and velocity are $K_I = 0.82 \approx K_x$, $K_v = 1.23 \approx 1/K_x$, and $K_t = 0.615 \approx K_x^2$, respectively. The calculated scale factors according to (14) could agree quite well with the test data. We can see that the scaling method is feasible.

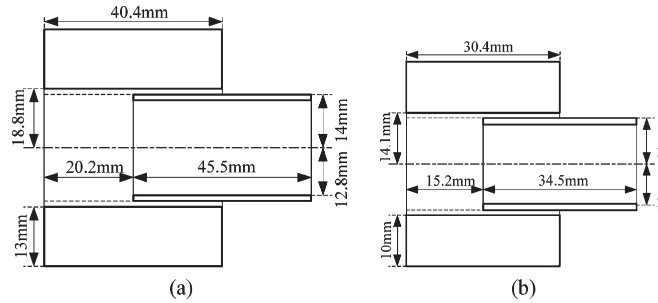


Fig. 3. Geometry parameters of the coil-gun experiment. (a) Geometry of the original. (b) Geometry of the model.

TABLE I
SCALING RESULTS OF THE EXPERIMENTS

	Original	model	Scaling factor	
			Input	Tested
geometry	——	——	0.8	——
mass of the armature	40g	20g	2	——
capacitor	1920uF	960uF	2	——
Initial voltage	500V	620V	1.24	——
current peek	4.5kA	3.7kA	——	0.82
velocity	4.13m/s	5.07 m/s	——	1.23
rise time of the current	260us	160us	——	0.615

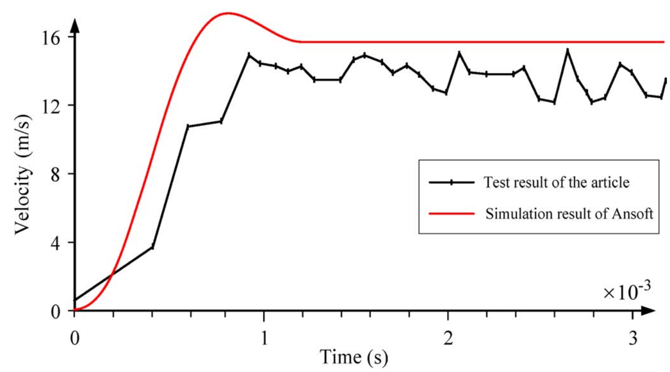


Fig. 4. Comparison of the test result and Ansoft result.

IV. Numerical Validation

There is only one stage coil in the experiment which cannot demonstrate the changes in velocity with respect to the discharge position in a multistage coil gun nor show us the field results. Thus, two 3-D finite-element models based on Ansoft were constructed to validate the scaling relationships.

The transient solver in Ansoft has been widely used in coil-gun studies [9], [10]. To confirm the accuracy of Ansoft, the experiment parameters in [11] are simulated and compared. The test result in [11] and the Ansoft simulation result are shown in Fig. 4. It is shown that the simulation result is a bit higher than the test results. However, taking friction, noncoaxiality, and other factors into consideration, the simulation result is thought to be correct, and Ansoft is workable to solve the coaxial induction coil gun.

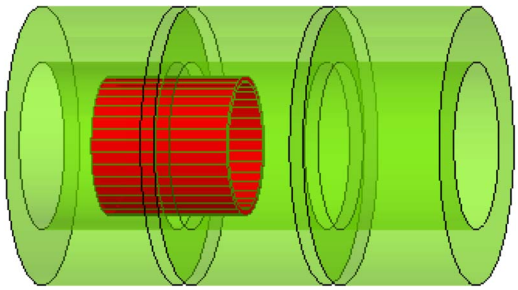


Fig. 5. 3-D simulation model of the three-stage coil gun based on Ansoft.

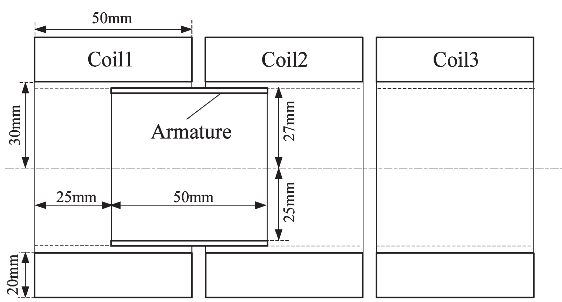


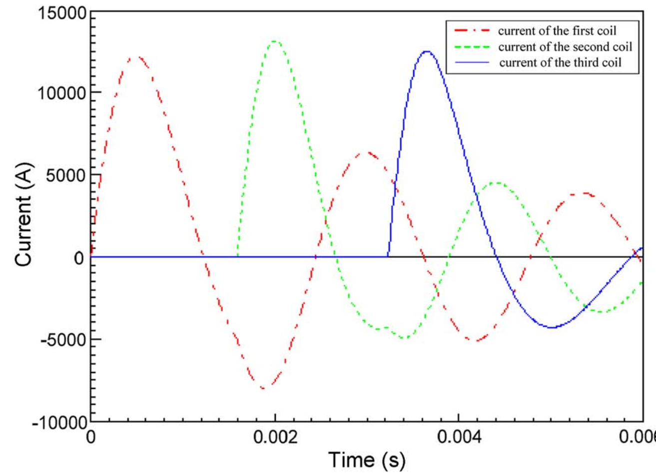
Fig. 6. Geometry parameters of the original coil-gun simulation.

For simplicity, the power sources of every stage coil were set to be the same. Every coil in the original is powered by a capacitance of 2 mF and initial voltage of 1.5 kV. According to (15), every coil in the model is powered by a capacitance of 0.25 mF and initial voltage of 6 kV. The mass of the original armature is 0.2 kg, and the mass of the armature model is 0.1 kg.

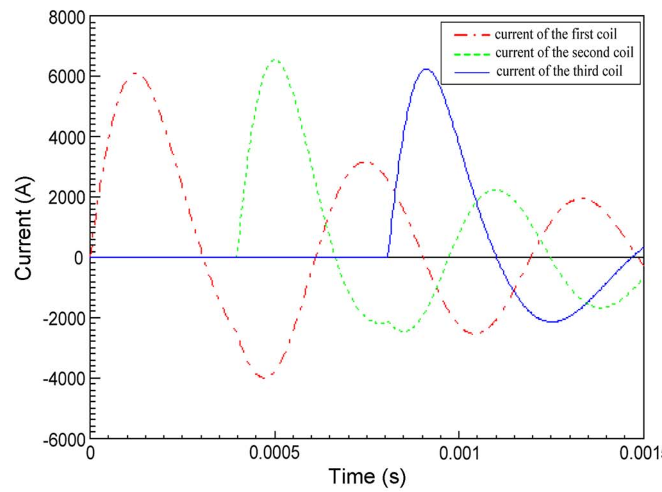
The original and the model in the simulation look exactly the same, as shown in Fig. 5. The geometry of the original is shown in Fig. 6. The geometric scale factor K_x and voltage scale factor K_{U0} are selected to be the input scale factors, where $K_x = 0.5$ and $K_{U0} = 4$. According to the scaling relationships of (14), the output scale factors can be calculated as

$$\begin{cases} K_{C_i} = 0.125 \\ K_t = 0.25 \\ K_{m_0} = K_M = K_L = K_r = K_{r_0} = K_x = 0.5 \\ K_\eta = K_\rho = K_\mu = K_\sigma = 1 \\ K_v = K_R = K_{I_{ck}} = K_{I_{aj}} = K_{I_{ak}} = K_{I_{ci}} = 2 \\ K_B = K_{U_i} = K_{U_0} = 4 \\ K_J = 8. \end{cases} \quad (15)$$

The simulation results of the original and the model are shown in Figs. 7–9 respectively. It is seen that the calculated scale factors could satisfy (15) quite well, where $K_I = K_x = 0.5$, as shown in Fig. 6, and $K_v = 1/K_x = 2$ and $K_t = K_x^2 = 0.25$ as shown in Fig. 7. The magnetic-flux densities at the corresponding moment of the two models are shown in Fig. 8. It is shown that the magnetic-flux density distribution in the two systems are the same, while their values fulfill $B'/B = K_B = K_x^2 = 4$. The simulation results show quite clearly that every parameter follows the scaling relationships of (15).



(a)



(b)

Fig. 7. Simulation results of the current curves. (a) Current curves of the original. (b) Current curves of the model.

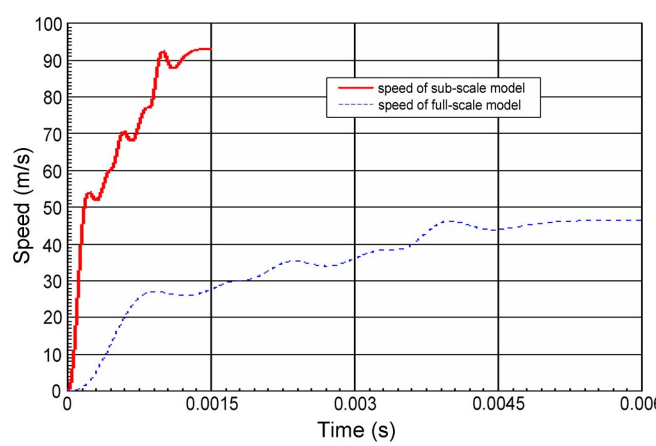
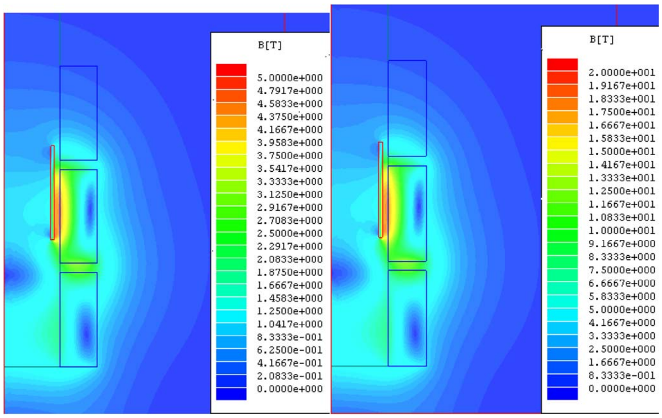


Fig. 8. Speed curves of the two models.

V. CONCLUSION

The application of the scaling method as a scientific method has a long tradition in some research areas. Since experiments are very important in the coil-gun research, the scaling method can also be a valuable aid here. The scaling relationships of a capacitor-driven coil gun have been deduced in this paper.



(a) (b)

Fig. 9. Magnetic-flux density of the two models. (a) The magnetic-flux density of the original at 0.002 s. (b) The magnetic-flux density of the model at 0.0005 s.

Then, two single-stage coil guns were constructed to verify the conclusion. Further 3-D finite-element simulations were constructed to verify the scale factors of velocity and field quantity in the multistage coil gun. Analytical conclusions are summarized as follows. Scaling method is feasible in the coil-gun design. With proper application, scaling relationships allow the construction of experimental models in which phenomena similar to those occurring in the original are reproduced. The results can then be recalculated, using the scale factor for the physical quantity, into the original configuration. The efficiencies of the two systems are the same. The scale factors can be selected flexibly based on the different experiment conditions.

REFERENCES

[1] W. Ying, R. A. Marshall, and C. Shukang, *Physics of Electric Launch*. Beijing, China: Science Press, 2004.

[2] M. S. Aubuchon, T. R. Lockner, R. J. Kaye, and B. N. Turman, "Study of coilgun performance and comments on powered armatures," in *Proc. IEEE Int. Power Modulator Conf.*, May 2004, pp. 141–144.

[3] S. Williamson and A. Smith, "Pulsed coilgun limits," *IEEE Trans. Magn.*, vol. 33, no. 1, pp. 201–207, Jan. 1989.

[4] K. McKinney and P. Mongeau, "Multiple stage pulsed induction acceleration," in *Proc. 2nd Symp. Electromagn. Launch Technol.*, Oct. 1983, p. 44.

[5] K.-T. Hsieh and B. Kim, "One kind of scaling relations on electromechanical systems," *IEEE Trans. Magn.*, vol. 33, no. 1, pp. 240–244, Jan. 1997.

[6] H. D. Yun, "EM gun scaling relationships," *IEEE Trans. Magn.*, vol. 35, no. 1, pp. 484–488, Jan. 1999.

[7] S. Satapathy and H. Vanicek, "Energy partition and scaling issues in a railgun," *IEEE Trans. Magn.*, vol. 43, no. 1, pp. 178–185, Jan. 2007.

[8] J. L. He, E. Levi, Z. Zabar, and L. Birenbaum, "Concerning the design of capacitively driven induction coil guns," *IEEE Trans. Plasma Sci.*, vol. 17, no. 3, pp. 429–438, Jun. 1989.

[9] Z. Keyi, L. Bin, L. Zhiyuan, C. Shukang, and Z. Ruiping, "Inductance computation consideration of induction coil launcher," *IEEE Trans. Magn.*, vol. 45, no. 1, pp. 336–340, Jan. 2009.

[10] Z. Keyi, C. Shukang, and Z. Ruiping, "Influence of driving current's wave on accelerative performance of induction coil launcher," in *Proc. Electromagn. Launcher Technol.*, Jun. 2008, pp. 1–4.

[11] P. J. Leonard, H. C. Lai, G. Hainswort, H. D. Rodger, and J. F. Eastham, "Analysis of the performance of tubular pulsed coil induction launchers," *IEEE Trans. Magn.*, vol. 29, no. 1, pp. 686–690, Jan. 1993.

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