Autonomous Intelligent Charging/Discharging of Electric Vehicles using Distributed Multi-Agent ADMM Framework for Grid Ancillary Services

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1 Abstract

The increasing popularity of Electric Vehicles (EVs) in the distribution grid along with technological advancement in EV electronics such as vehicle to grid (V2G) technique has enabled them to participate in grid ancillary services. To achieve this, the EVs need to establish a contract with third-party aggregators and connect to a charging unit, either residential or commercial. At any time they are connected, the EVs can decide to take part in the ancillary services program offered to them by the aggregators. If agreed, the aggregators will use the EVs as a power source capable of charging/discharging power according to the input signal, and in return, they will be compensated. This inter-temporal nature of charging/discharging is also transforming the traditional optimal power flow (OPF) problem into a dynamic OPF problem. This chapter aims at developing a multi-layer time-dependent optimization algorithm to utilize EV potential and provide ancillary services while maximizing its utilization function. Specifically, in the upper layer, an autonomous distributed ADMM algorithm is developed to optimize the cost for charging/discharging EVs while using them to regulate the voltage at each bus in the distribution grid. The distributed ADMM algorithm is also expanded to the lower layer where the individual EVs active and reactive power is controlled for voltage regulation while maintaining the desired state of the charge of the vehicle at the end of the charging period. The effectiveness and performance improvement of the proposed multi-layer algorithm is illustrated through analytical analysis and simulation results.

2 Introduction

Over the last decade, we have witnessed a massive surge of Electric Vehicle (EV) users in our community. Regular working people are buying EV as their primary vehicle due to the push from the government for a "green" earth and reduce greenhouse gas emissions. It became more lucrative when leading automotive industries paid their attention to developing charging infrastructure, upgrading battery capacity, and have vehicle models at a reasonable price [1]. Due to these reasons, EV sales in the United States have tripled between 2014 and 2018 [2]. As a consequence, there is a large number of EVs connected to the distribution grid which is causing substantial degradation in efficiency and reliability of the distribution power grid [3]. The authors in [4] and [5] showed that the charging of a large number of EVs at the same time puts additional strain on the grid and contributes to many negative effects such as increased distribution energy losses, voltage deviation, and overload of distribution network lines and substations. But not all are bad with EVs in the distribution grid as EVs can be used as a valuable resource that can be controlled to benefit the gird [6]. We can use EVs as a flexible grid-connected energy resource and to effectively use its full potential, EVs can be controlled to not only charge but also to discharge and return energy to the grid [7]. To this end, we will develop a multi-layer distributed Alternating Direction Method of Multiplier (ADMM) algorithm which can handle dynamic EV constraints like the state of charge (SOC) to optimize and control EVs in the grid to properly utilize them for voltage regulation and at the same time, ensure that they are compensated by maximizing their utility function and have desired state of charge at the end of their charging period. To analyze multi-layer distributed ADMM, one must understand the basics of original ADMM which is presented in section 2. In section 3, we introduce the developed distributed ADMM algorithm and discuss its convergence properties. In section 4, we formulate a problem of voltage deviation minimization and EV utility maximization for a distribution system network with EV penetration and implement the proposed algorithm and present some simulation results to illustrate the results. And finally, we conclude the chapter in section 5.

3 Alternating Direction Methods of Multipliers: An Overview

Let us consider the problem below [8]

$$\begin{aligned} \min_{x,z} \quad & f(x) + g(z) \\ s.t. \quad & Ax + Bz = c \end{aligned} \tag{1}$$

where variables $x \in \mathcal{R}^n$ and $z \in \mathcal{R}^m$. $A \in \mathcal{R}^{p\times n}$, $B \in \mathcal{R}^{p\times m}$ and $c \in \mathcal{R}^p$ are the matrices and vector in the linear constraint. We will assume f and g are convex and differentiable and that their gradients are locally Lipschitz. The problem can be thought of as a general convex linear equality-constrained problem except for the fact that the main optimization variable has been split into two parts, namely x and z in this case, with objective function separable across this splitting, and thus the algorithm is distributed in nature. As with any primal-dual convex optimization, we form the Lagrangian as follows

$$L_\rho(x,z,y) = f(x) + g(z) + y^T(Ax + Bz - c) + \frac{\rho}{2}||Ax + Bz - c||_2^2. \quad (2)$$

The Lagrangian in this case is called the "Augmented Lagrangian" due to the fact that an additional penalty term with multiplier $\rho > 0$ is added to the objective function. The problem is solved using ADMM with the following iterations:

$$x^{k+1} := \arg\min_x L_\rho(x, z^k, y^k) \quad (3a)$$

$$z^{k+1} := \arg\min_z L_\rho(x^{k+1}, z, y^k) \quad (3b)$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \quad (3c)$$

The ADMM algorithm consists of a x-minimization step (3a), followed by a z-minimization step (3b) and a dual variable update (3c). The primal variables x and z are updated in an alternating fashion thus the name "alternating direction". The ADMM algorithm of this type can only deal with static linear constraints but falls short when the constraint becomes dynamic in nature. This is very true when we deal with a distribution network with EV penetration since EV itself has a State of Charge (SOC) parameter which is dynamic in nature and evolves with time. To tackle the problem of this nature, in the next section, we will develop our proposed ADMM algorithm with dynamic constraints in the continuous-time domain.

4 A Distributed Multi-Agent Networked ADMM

In this section of the chapter, we develop a continuous-domain real-time ADMM algorithm with a dynamic constraint that can be implemented to a broad class of networked multi-agent distributed optimization and control problems.

4.1 Networked Multi-Agent System

Let us consider a networked multi-agent system which is characterized by a bidirectional graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \cdots, N\}$ is the number of agents and $\mathcal{E}$ represents the set of edges between them. Also, let node 1 be the virtual leader of the network which means, through the communication network, node 1 can obtain the information of all the nodes in the network if necessary. We can define the network interconnection with the following binary matrix [9]:

$$S = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & s_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & s_{N2} & \cdots & 1 \end{bmatrix}, \quad (4)$$

where $s_{ij} = 1$ if and only if $(i \leftrightarrow j) \in \mathcal{E}$, and $s_{ij} = 0$ if otherwise. That means that if agent i can communicate with agent j and vice versa, we have an entry of 1 in the position (i, j) in the communication matrix. The matrix S has 1 in the diagonal as every agent knows its information. We assume that the communication matrix S is irreducible, i.e., the communication graph is strongly connected.

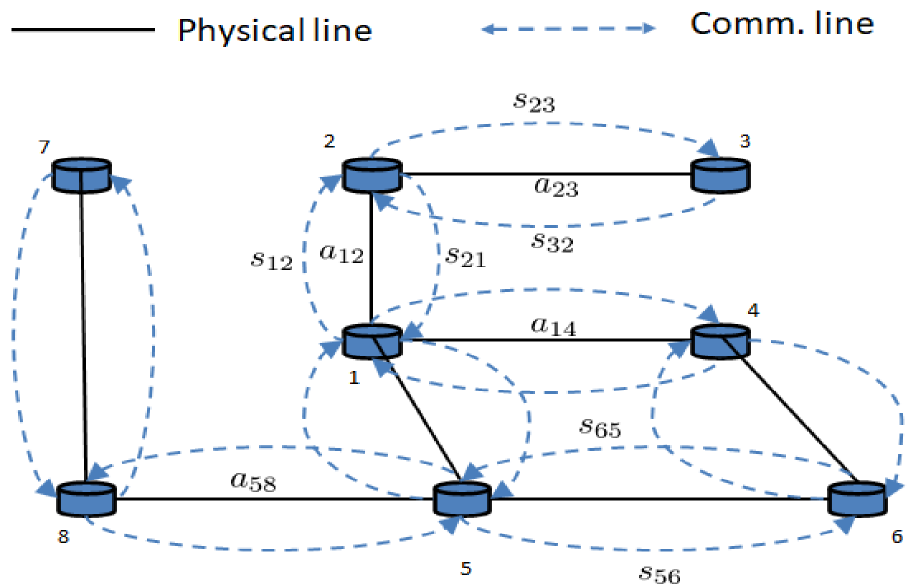


Figure 1: A networked multi-agent system with both physical and communication layer

Figure 1 shows the network of multi-agent system. The agents can be connected physically as we will see in the next subsection and it will also be required when we implement the algorithm in a power distribution system.

4.2 Distributed Dynamic ADMM

In this subsection, we will formulate a distributed networked multi-agent problem with a dynamic constraint. The work in [10] formulated a similar kind of problem without the dynamic constraint and the solution with convergence proof was provided in the discrete-time domain. In this chapter, we will add a dynamic constraint and tackle the problem in the continuous-time domain.

Consider the following distributed optimization problem:

$$\min_{y_i} \sum_{i \in \mathcal{N}} f_i(y_i)$$

s.t.

$$\sum_{j\in\mathcal{N}_i} A_{ij}z_{ji} = 0, \quad \forall i \in \mathcal{N}; \quad y_i = z_{ij}, \quad \forall i \in \mathcal{N},\ j \in \mathcal{N}_i,$$

$$\dot{x}_i = F_i(x_i) + G_i(x_i)u_i, \quad y_i = H_i(x_i),$$

where

- $x_i \in \mathcal{R}^{n_i}$ is the state of the ith agent
- $y_i \in \mathcal{R}^{l_i}$ is the output of the ith agent
- $f_i(y_i) : \mathcal{R}^{l_i} \to\in \mathcal{R}$ is the objective function of agent i
- $u_i \in \mathcal{R}^{m_i}$ is the control (or decision variable) vector.
- A_{ij} are constant matrices of appropriate dimension.

We assume that the functions $f_i(y_i)$ are convex and differentiable, and their gradients are locally Lipschitz. In the above problem, y and z are the two primal variables used in the original ADMM. The matrix A_{ij} represents the physical interconnection between the agents as shown in figure 1. To tackle the problem, let us define the following design principles:

(i) Let

$$u_i = \mathcal{U}_i(x_i) + \omega_i$$

where local feedback control $\mathcal{U}_i$ is designed so that subsystem of x_i is input-to-state stable and that, if $\omega_i \to c_i$ for any constant $c_i, y_i \to c_i$.

(ii) Input ω_i is chosen as the ADMM law.

(iii) Using the communication matrix S, let us also define a gain matrix D whose values are calculated according to the following equation:

$$D = [d_{ij}] \in \mathbb{R}^{N\times N}, \quad d_{ij} = \frac{s_{ij}\beta_{ij}}{\sum_{l=1}^{N} s_{il}\beta_{il}}, \tag{5}$$

where $\beta_{ij} > 0$ are piecewise-constant scalar gains. The matrix D is a non-negative, row stochastic and diagonally positive matrix.

With the above design principle, we can redefine the optimization problem as follows:

$$\min_{\omega_i} \quad \sum_{i\in\mathcal{N}} f_i(y_i) \tag{6a}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{N}_i} A_{ij}z_{ji} = 0 \qquad j \in \mathcal{N}_i, i \in \mathcal{N} \tag{6b}$$

$$y_i = z_{ij} \qquad j \in \mathcal{N}_i, i \in \mathcal{N} \tag{6c}$$

$$\dot{x}_i = F_i(x_i) + G_i(x_i)u_i \tag{6d}$$

$$u_i = \mathcal{U}_i(x_i) + \omega_i \tag{6e}$$

Now we can form the so called augmented Lagrangian as follows:

$$L(u, z, \lambda) = \sum_{i \in \mathcal{N}} L_i(u_i, z_{ij}, \lambda_{ij}) \tag{7}$$

$$L_i = f_i(y_i) + \sum_{j \in \mathcal{N}_i} \left[d_{ij} \lambda_{ij}^T (y_i - z_{ij}) + \frac{d_{ij}}{2} ||y_i - z_{ij}||^2 \right]$$

It should be noted that only the consensus constraint (6c) is used in the augmented Lagrangian since it is the only constraint that contains both the primal variables. The rest of the constraints are taken into account while solving the individual sub-problems. Also, the penalty parameter term in the augmented Lagrangian is replaced with entries d_{ij} from the D matrix defined in the design principle [10]. This enables the penalty term to conform with the actual interconnection of the agents. Because of this reason, the dual variable λ is also scaled by the d_{ij} term. The augmented Lagrangian (7) is solved using ADMM by solving the following sub-problems in an alternating sequential manner:

1. For any $i \in \mathcal{N}$, ω_i is updated according to

$$\dot{\omega}_i = \underset{x_i \in \mathbb{R}^n}{\arg\min}\, L(y, z^-, \lambda) \tag{8}$$

2. For any $i \in \mathcal{N}$ and for $j \in \mathcal{N}_i$, z_{ji} is solved as

$$\dot{z}_{ji} = \underset{z_{ji} \in \mathbb{R}^n}{\arg\min} \quad L(y, z, \lambda) \tag{9}$$

$$\text{s.t.} \sum_{j \in \mathcal{N}_i} A_{ij} z_{ji} = 0 \tag{10}$$

3. For any $i \in \mathcal{N}$ and for $j \in \mathcal{N}_i$, λ_{ji} evolves as

$$\dot{\lambda}_{ji} = \underset{\lambda_{ji} \in \mathbb{R}^n}{\arg\max}\, L(y, z, \lambda) \tag{11}$$

where $z^- \triangleq z(t^-)$ is the immediate past solutions to the problems of (10). We use the delayed version of z to mimic the alternating behavior of the ADMM algorithm. Using the convex properties and techniques from [10], we obtain the continuous-time dynamics including dynamic constraints for the solution of sub-problems (8) - (11) as follows:

$$\dot{\omega}_i = -\alpha_i \left[\nabla_{y_i} f_i(y_i) + \sum_{j \in \mathcal{N}_i} d_{ji} \lambda_{ij} + \sum_{j \in \mathcal{N}_i} d_{ij} (\xi_i - z_{ij}^-) \right] \tag{12a}$$

$$\dot{z}_{ji} = -\alpha_i [-d_{ij} \lambda_{ji} - d_{ji} (\xi_j - z_{ji}) + A_{ij}^T \mu_i] \tag{12b}$$

$$\dot{\mu}_i = w_i \sum_{j \in \mathcal{N}_i} A_{ij} z_{ji} \tag{12c}$$

$$\dot{\lambda}_{ji} = d_{ji} (\xi_j - z_{ji}). \tag{12d}$$

where μ_i is the dual variable associated with the constraint (10) of z-minimization sub-problem. The variable ξ_i is replacing ω_i in all the dynamics equations which is to be designed using the passivity-short framework [11]. Let us also define the error states as $\tilde{e}_i = e_i - e_i^*$ where $e = \{u_i, z_{ij}, \lambda_{ij}, \mu_i, x_i, y_i\}$. With this, the error dynamics are given as

$$\dot{\tilde{\omega}}_i = -\alpha_i \left[(\nabla_{y_i} f_i(y_i) - \nabla_{y_i^*} f_i(y_i^*)) + \sum_{j\in\mathcal{N}_i} d_{ij}\tilde{\lambda}_{ij} + \sum_{j\in\mathcal{N}_i} d_{ij}(\tilde{\xi}_i - \tilde{z}_{ij}^{-}) \right] \quad (13a)$$

$$\dot{\tilde{z}}_{ji} = -\alpha_i[-d_{ji}\tilde{\lambda}_{ji} - d_{ji}(\tilde{\xi}_j - \tilde{z}_{ji}) + A_{ij}\tilde{\mu}_i] \quad (13b)$$

$$\dot{\tilde{\mu}}_i = w_i \sum_{j\in\mathcal{N}_i} A_{ij}\tilde{z}_{ji} \quad (13c)$$

$$\dot{\tilde{\lambda}}_{ji} = d_{ji}(\tilde{\xi}_j - \tilde{z}_{ji}). \quad (13d)$$

We present theorem 1 below which shows that the error state dynamic equations (12) obtained converges to the optimal solutions.

Theorem 1: *Consider the statically and dynamically constrained optimization problem* (6)*. Suppose that* $\mathcal{U}_i$ *is designed such that, for constant matrices* C_{il}*, there is an individual positive definite Lyapunov function* $V_i^{'}(x_i, \omega_i)$ *satisfying the following inequality:*

$$\left[\nabla_{x_i} V_i^{'}(x_i,\omega_i)\right]^T \left[F_i(x_i) + G_i(x_i)\mathcal{U}_i + G_i(x_i)\omega_i\right] + \left[\nabla_{\omega_i} V_i(x_i,\omega_i)\right]^T \dot{\omega}_i$$
$$\leq -\sum_l k_i||\omega_i - C_{il}x_i||^2 + \left[\sum_l (\omega_i - C_{il}x_i)\right]^T \dot{\omega}_i + (y_i - v_i)\dot{\omega}_i \quad (14)$$

Then, continuous-time ADMM algorithm (13) *with*

$$\xi_i = y_i + \sum_l (\omega_i - C_{il}x_i)$$

is globally convergent to the optimal solution.

The proof of the theorem is provided in the Appendix at the end of the chapter. Using the set of dynamic equations (12), we will be solving the distribution grid problem with EV penetration in the following section.

5 Dynamic ADMM Application

In our proposed approach, we have divided the distribution system into three separate layers. On the top layer is the Distribution System Operator (DSO) who is in charge of the overall distribution grid. The DSO is responsible for maintaining optimal power flow in the grid as well as maintaining system parameters like voltage within the desired tolerable range. In the next layer, we assume the existence of a third party entity, generally termed as the aggregator in the literature, which collects EV information at each node and can also send/receive input/output signals. And in the final layer, we have all the consumers with the EVs who agree and signs a contract with the aggregators, allowing them to use the EVs for grid services in exchange for financial compensation. We assume at any instance of time, if the EV owners permit through the contract, aggregators can gather information on EV's active and reactive power injection at each bus through the existing sensor network between them in real-time. Figure 2 shows the structure of the multi-layer distribution grid and the associated variables which are defined in the next sub-section.

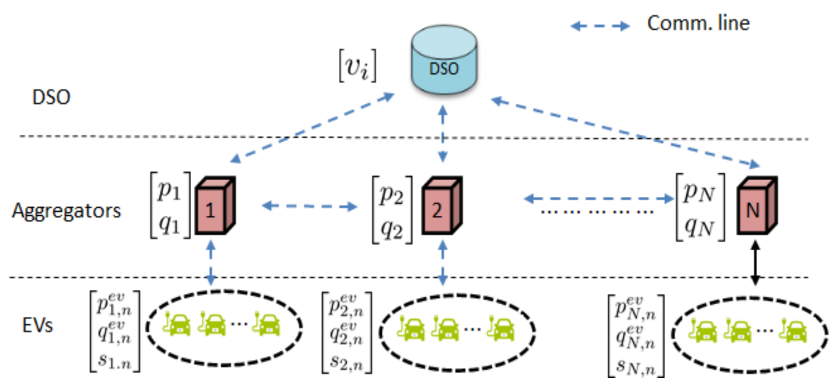


Figure 2: The multi-layer representation of distribution grid with associated variables

5.1 Branch Flow Model

For the branch flow model, consider a radial distribution network by a directed graph $G = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} := \{1, \cdots, N\}$ represents the set of buses and $\mathcal{E}$ represents the set of distribution lines connecting the buses in $\mathcal{N}$. Without any loss of generality, the substation of the radial network is indexed by 1. Each node $i \in \mathcal{N} \setminus 1$ has an unique parent node Γ_i and a set of children nodes, denoted by $\mathcal{C}_i$ as shown in figure 3. We assume each directed line points towards its children, i.e., power flows from parent Γ_i to node i.

For each bus $i \in \mathcal{N}$, let $\mathcal{N}_i$ bet the set of neighboring nodes including node i and its parent, i.e., $\mathcal{N}_i = \Gamma_i \cup \{i\} \cup \mathcal{C}_i$. Also, let V_i be its voltage with $P_{d_i} + jQ_{d_i}$ being the load demand and $p_i + jq_i$ is the aggregated active and reactive power injection by the EVs. For the branch $\Gamma_i \rightarrow i$, let I_i be the current flowing through it with $R_{\Gamma_i i} + jX_{\Gamma_i i}$ being the impedance of the line and $P_{\Gamma_i i} + jQ_{\Gamma_i i}$ being the complex power flowing from the parent Γ_i to node i. For bus $i \in \mathcal{N}$,

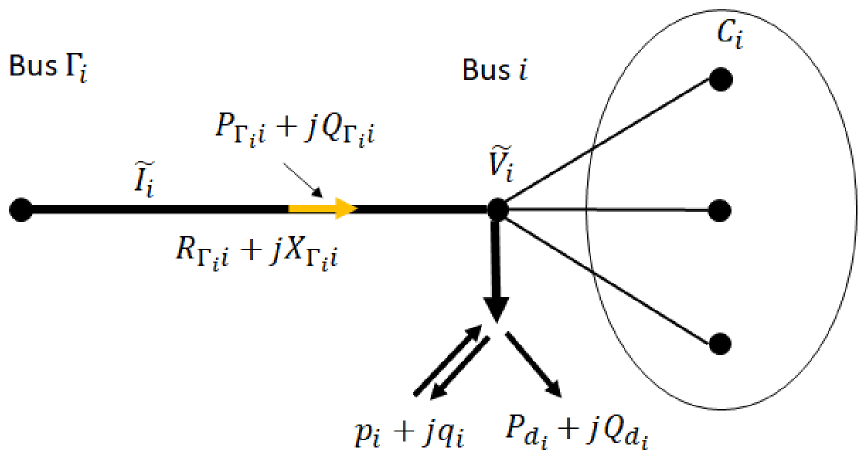


Figure 3: A distribution network.

the power balance equations are given as

$$P_{\Gamma_i i} = p_i + P_{d_i} + \sum_{j \in \mathcal{C}_i} (P_{ij} + R_{ij} l_{ij}) \qquad i \in \mathcal{N} \tag{15a}$$

$$Q_{\Gamma_i i} = q_i + Q_{d_i} + \sum_{j \in \mathcal{C}_i} (Q_{ij} + X_{ij} l_{ij}) \qquad i \in \mathcal{N} \tag{15b}$$

where $l_{ij} = I_i^2$ is the square of current magnitude which is defined as

$$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \tag{16}$$

where $v_i = V_i^2$ is the square of the voltage magnitude. Equation (16) is a non-linear equation which can be linearized around current operating point P_{ij}^o, Q_{ij}^o, v_i^o and l_{ij}^o as below:

$$l_{ij} - l_{ij}^o = \frac{2P_{ij}^o}{v_i^o}(P_{ij} - P_{ij}^o) + \frac{2Q_{ij}^o}{v_i^o}(Q_{ij} - Q_{ij}^o) - \frac{(P_{ij}^o)^2 + (Q_{ij}^o)^2}{(v_i^o)^2}(v_i - v_i^o) \tag{17}$$

The power flow on all lines $(i, j) \in \mathcal{E}$ are expressed as

$$v_i - v_j = 2(R_{ij} P_{ij} + X_{ij} Q_{ij}) + (R_{ij}^2 + X_{ij}^2) l_{ij} \tag{18}$$

5.2 EV Parameters

In our structure, EV owners, who are interested to sign up for grid services, are connected to aggregators at each node. Thus, The aggregated power of all EVs connected to node i at time t is

$$p_i(t) = \sum_{n=1}^{E_i} p_{i,n}^{ev}, \qquad \forall\, i \in \mathcal{N} \tag{19}$$

where E_i denotes the number of EVs controlled by the aggregator in charge of bus i and $p_{i,n}^{ev} \in \{0, \overline{p}_{i,n}^{ev}, \underline{p}_{i,n}^{ev}\}$ is the charging/discharging power of EV n. Let us denote the time of arrival of nth EV as $t_{a,n}$ and departure time as $t_{d,n}$. The EV owner registers this time with the aggregator so that they can be used in the continuous-domain real-time optimization framework. The nth EV only charge/discharge itself between this time-frame. Thus we can write:

$$p_{i,n}^{ev} = \begin{cases} 0 & t \notin [t_{a,n}, t_{d,n}] \\ \left[\overline{p}_{i,n}^{ev}, \underline{p}_{i,n}^{ev}\right] & t \in [t_{a,n}, t_{d,n}] \end{cases} \tag{20}$$

The state of charge dynamics $s_{i,n}$ of EV can be represented by the following first order differential equation:

$$\dot{s}_{i,n} = \frac{\eta_n p_{i,n}^{ev}}{B_{i,n}} \tag{21}$$

At the end of the charging period, the EV consumers wants their EVs to be charged to a specific state of charge, specifically

$$s_{i,n}(t_{d,n}) = s_{i,n}^{d}, \qquad n \in \mathcal{E}_i,\ i \in \mathcal{N} \tag{22}$$

where $s_{i,n}^{d}$ is the desired SOC.

5.3 Objective Functions

For the Distribution System Operator (DSO), the objective would be to minimize the generation cost and maintain the voltage close to unity at every bus over the time horizon, mathematically:

$$f_i(p_i, v_i) = C_i(p_i) + H_i(v_i) \qquad i \in \mathcal{N} \tag{23}$$

where $C_i(p_i(t))$ is the cost function for generation production and $H_i(v_i(t))$ is the voltage regulation penalty function.

As for the EV owners, the want to minimize their charging cost and maximize their utility. Thus we can express their welfare function as follows

$$W_{i,n}(p_{i,n}^{ev}) = \psi_{i,n} p_{i,n}^{ev} - U_i(p_{i,n}^{ev}) \qquad n \in \mathcal{E}_i,\ i \in \mathcal{N} \tag{24}$$

We also have a terminal condition, where at the end of a charging period, the SOC of the EV should be at the desired SOC, that is

$$S_{i,n}(s_{i,n}^{d}) = k_{i,n}(s_{i,n} - s_{i,n}^{d})^2 \tag{25}$$

where $k_{i,n} > 0$ is the weight on terminal condition.

5.4 Problem Formulation

Let us stack all the state variables into a vector ω_i, that is

$$\omega_i = [v_i, p_i, q_i, P_{\Gamma_i i}, Q_{\Gamma_i i}, l_{\Gamma_i i}, p^{ev}_{i,n}, q^{ev}_{i,n}, s_{i,n}]^T \qquad \forall n = [1, \cdots, E_i],\ i \in \mathcal{N}$$

we also introduce an observation vector z_{ji} which represents the variables of node j at node i, that is $z_{ji} = [v^i_j, p^i_j, q^i_j, P^i_{\Gamma_j j}, Q^i_{\Gamma_j j}, l^i_{\Gamma_j j}, p^{ev,i}_{j,n}, q^{ev,i}_{j,n}, s^{ev,i}_{j,n}]^T$. With these definitions, we can formulate our optimization problem according to the developed dynamic ADMM as follows :

$$\min_{\omega_i} \quad \sum_{i=1}^{N}\sum_{n=1}^{E_i}\Big[\phi_i(\omega_i)\Big] \tag{26a}$$

$$\tag{26b}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{N}_i} A_{ij} z_{ji} + m_{ij} = 0 \qquad \forall i \in \mathcal{N} \tag{26c}$$

$$\dot{x}_i = A_i(x_i) + B_i(x_i)u_i \tag{26d}$$

$$y_i = z_{ij} \tag{26e}$$

$$y_i = \omega_i \tag{26f}$$

$$u_i = \mathcal{U}_i(x_i) + \omega_i \tag{26g}$$

where $\phi_i(\omega_i) = \Big[S_{i,n}(s_{i,n}) + f_i(p_i, v_i) + W_{i,n}(p^{ev}_{i,n})\Big]$, $x_i = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ s_i]^T$ and $\mathcal{U}_i(x_i) = 1$. It should be noted that the objective function is not summed over time since the problem is solved in real time with continuous-domain dynamics where the arrival and departure time can be tackled by the individual EV dynamics. Based on what agent j represents, the matrix A_{ij} and vector m_{ij} takes the following form:

$$A_{ii} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 1 & 0 & 0 & 2R_{\Gamma_i i} & 2X_{\Gamma_i i} & (R^2_{\Gamma_i i} + X^2_{\Gamma_i i}) & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & \frac{2P^o_{\Gamma_i i}}{v^o_{\Gamma_i}} & \frac{2Q^o_{\Gamma_i i}}{v^o_{\Gamma_i}} & -1 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \end{bmatrix},$$

$$A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & R_{ij} & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & X_{ij} & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \end{bmatrix}, \quad j \in \mathcal{C}_i,$$

$$A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \\ -\frac{(P^o_{ij})^2 + (Q^o_{ij})^2}{(v^o_{\Gamma_i})^2} & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 & 0 & \cdot & 0 \end{bmatrix}, \quad j = \Gamma_i.$$

and $m_{ii} = [P_{d_i}\ Q_{d_i}\ 0\ 0]^T$, $m_{ij} = \{[0\ 0\ 0\ 0]^T, j \in \mathcal{C}_i\}$ and $m_{ij} = \left\{\left[0\ 0\ 0\ \left(-\frac{2(P^o_{ji})^2 v^o_j + 2(Q^o_{ji})^2 v^o_j + (P^o_{ji})^2 - (Q^o_{ji})^2}{(v^o_j)^2} - l^o_{ji}\right)\right]^T, j \in \Gamma_i\right\}$.

Following the procedures developed in section 2 and dynamic equation set (12), we obtain the following update dynamics for the system

$$\dot{v}_i = -\alpha_i \left[\nabla_{v_i}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(1) + (v_i - v_i^{j-})\right]\right] \tag{27a}$$

$$\dot{p}_i = -\alpha_i \left[\nabla_{p_i}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(2) + (p_i - p_i^{j-})\right]\right] \tag{27b}$$

$$\dot{q}_i = -\alpha_i \left[\nabla_{q_i}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(3) + (q_i - q_i^{j-})\right]\right] \tag{27c}$$

$$\dot{P}_{\Gamma_i i} = -\alpha_i \left[\nabla_{P_{\Gamma_i i}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(4) + (P_{\Gamma_i i} - P_{\Gamma_i i}^{j-})\right]\right] \tag{27d}$$

$$\dot{Q}_{\Gamma_i i} = -\alpha_i \left[\nabla_{Q_{\Gamma_i i}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(5) + (Q_{\Gamma_i i} - Q_{\Gamma_i i}^{j-})\right]\right] \tag{27e}$$

$$\dot{l}_{\Gamma_i i} = -\alpha_i \left[\nabla_{l_{\Gamma_i i}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{ij}(6) + (l_{\Gamma_i i} - l_{\Gamma_i i}^{j-})\right]\right] \tag{27f}$$

$$\dot{p}^{ev}_{i,n} = -\alpha_i \left[\nabla_{p^{ev}_{i,n}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{in}(7) + (p^{ev}_{i,n} - p^{ev,j-}_{i,n})\right]\right] \tag{27g}$$

$$\dot{q}^{ev}_{i,n} = -\alpha_i \left[\nabla_{q^{ev}_{i,n}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{in}(8) + (q^{ev}_{i,n} - q^{ev,j-}_{i,n})\right]\right] \tag{27h}$$

$$\dot{s}_{i,n} = -\alpha_i \left[\nabla_{s_{i,n}}\phi_i(\omega_i) + \sum_{j\in\mathcal{N}_i} d_{ij}\left[\lambda_{in}(9) + (s_{i,n} - s^{j-}_{i,n})\right]\right] \tag{27i}$$

where $n \in [1, \cdots, E_i]$, $j \in \mathcal{N}_i$ and $i \in \mathcal{N}$. The z_{ji} is given as

$$\begin{bmatrix} \dot{v}_j^i \\ \dot{p}_j^i \\ \dot{q}_j^i \\ \dot{P}_{\Gamma_j j}^i \\ \dot{Q}_{\Gamma_j j}^i \\ \dot{l}_{\Gamma_j j}^i \\ \dot{p}_{j,n}^{ev,i} \\ \dot{q}_{j,n}^{ev,i} \\ \dot{s}_{j,n}^i \end{bmatrix} = \alpha_i d_{ji} \begin{bmatrix} \lambda_{ji}(1) \\ \lambda_{ji}(2) \\ \lambda_{ji}(3) \\ \lambda_{ji}(4) \\ \lambda_{ji}(5) \\ \lambda_{ji}(6) \\ \lambda_{jn}(7) \\ \lambda_{jn}(8) \\ \lambda_{jn}(9) \end{bmatrix} + d_{ji} \begin{bmatrix} v_j - v_j^i \\ p_j - p_j^i \\ q_j - q_j^i \\ P_{\Gamma_j j} - P_{\Gamma_j j}^i \\ Q_{\Gamma_j j} - Q_{\Gamma_j j}^i \\ l_{\Gamma_j j} - l_{\Gamma_j j}^i \\ p_{j,n}^{ev} - p_{j,n}^{ev,i} \\ q_{j,n}^{ev} - q_{j,n}^{ev,i} \\ s_{j,n} - s_{j,n}^i \end{bmatrix} - A_{ij}^T \begin{bmatrix} \mu_i(1) \\ \mu_i(2) \\ \mu_i(3) \\ \mu_i(4) \\ \mu_i(5) \\ \mu_i(6) \\ \mu_i(7) \\ \mu_i(8) \\ \mu_i(9) \end{bmatrix}$$

The dual updates are given as

$$\begin{bmatrix} \dot{\mu}_i(1) \\ \dot{\mu}_i(2) \\ \dot{\mu}_i(3) \\ \dot{\mu}_i(4) \\ \dot{\mu}_i(5) \\ \dot{\mu}_i(6) \\ \dot{\mu}_i(7) \\ \dot{\mu}_i(8) \\ \dot{\mu}_i(9) \end{bmatrix} = \sum_{j \in \mathcal{N}_i} A_{ij} \begin{bmatrix} v_j^i \\ p_j^i \\ q_j^i \\ P_{\Gamma_j j}^i \\ Q_{\Gamma_j j}^i \\ l_{\Gamma_j j}^i \\ p_{i,n}^{ev,j} \\ q_{i,n}^{ev,j} \\ s_{i,n}^j \end{bmatrix}$$

$$\begin{bmatrix} \dot{\lambda}_{ji}(1) \\ \dot{\lambda}_{ji}(2) \\ \dot{\lambda}_{ji}(3) \\ \dot{\lambda}_{ji}(4) \\ \dot{\lambda}_{ji}(5) \\ \dot{\lambda}_{ji}(6) \\ \dot{\lambda}_{jn}(7) \\ \dot{\lambda}_{jn}(8) \\ \dot{\lambda}_{jn}(9) \end{bmatrix} = d_{ji} \begin{bmatrix} v_j - v_j^i \\ p_j - p_j^i \\ q_j - q_j^i \\ P_{\Gamma_j j} - P_{\Gamma_j j}^i \\ Q_{\Gamma_j j} - Q_{\Gamma_j j}^i \\ l_{\Gamma_j j} - l_{\Gamma_j j}^i \\ p_{j,n}^{ev} - p_{j,n}^{ev,i} \\ q_{j,n}^{ev} - q_{j,n}^{ev,i} \\ s_{j,n} - s_{j,n}^i \end{bmatrix}$$

These dynamics were implemented on the IEEE 123 bus distribution system in a co-simulation with Matlab and OpenDSS. The following tables show the EV type and EV charging parameter used in the simulation.

In the simulation of the proposed algorithm on the distribution system, we defined $C_i(p_i) = a_i^p p_i^2$, $H_i(v_i) = (1 - v_i)^2$ and $U_i(p_{i,n}^{ev}) = a_{p_n} log(p_{i,n}^{ev} + 1)$ and the parameters of the coefficients were taken from [12]. The price of electricity was fixed at an average of 15 cents per kWh. We randomly placed 150 EVs among 40 buses in the system and we fixed that by the end of their charging

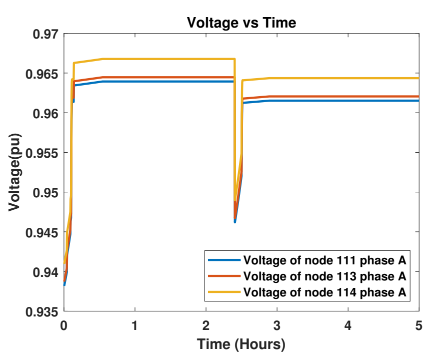
Table 1: EV charging categories

Charging method	Voltage	Max. current	Input power
AC level 1	120 V	12 A	1.4 kW
AC level 2	208 - 240 V	32 A	7.2-19.2 kW
DC charging	400 - 1000 V	300 A	50-150 kW

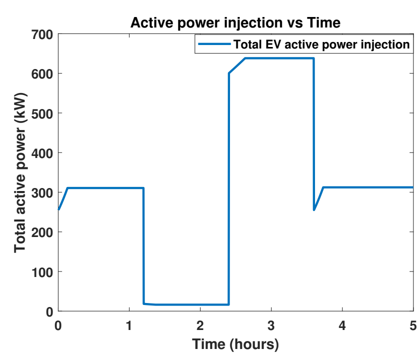
Table 2: EV Types

Brand name	Battery capacity
Nissan Leaf	40-62 kWh
Toyota RAV4-EV	41.8 kWh
BMW i3	42.2 kWh
Tesla model 3	75 kWh
Tesla model S	100 kWh
Tesla model X	100 kWh

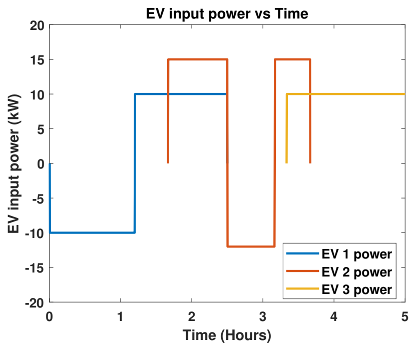
period, the EV owners want their vehicles to be charged to at least 90% of its capacity. The value of step-size α_i was fixed at 0.01 for all the nodes. We first ran a base case where the algorithm was not implemented and figured that bus 111,113 and 114 had the lowest voltage in the whole grid. So we observed the voltages of those 3 nodes after implementing the algorithm and the results are shown in figure 4a. Figure 4b shows the total amount of active power injection by the EVs over 5 hours responding to the voltage fluctuations in the grid. We selected 3 random EVs around node the three selected nodes and plotted their active power injection and how their SOC evolved which are presented in figure 5a and 5b, respectively. The figures shows the arrival and departure time of the EVs and we can observe that the EVs react to the voltage fluctuations in the grid and eventually at the end of their charging time, they get their SOC above 90%.



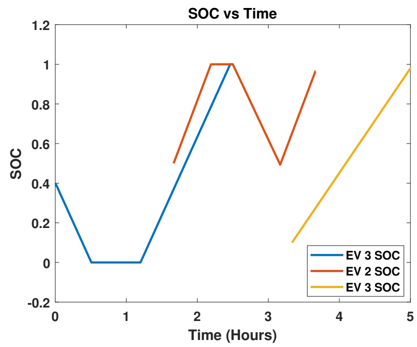
(a) Voltage at Node 111,113 and 114



(b) Total active power injection by EVs into the grid



(a) Active power injection by 3 EVs



(b) The SOC evolution of the 3 EVs

6 Conclusion

This chapter presents a novel continuous-domain distributed multi-agent ADMM algorithm with dynamic constraint. In contrast to usual discrete-time iterative solution techniques where the accuracy and the optimality of the solution depend on sampling and convergence time, the proposed algorithm can solve the problem in real-time. It is also capable of handling dynamic constraints that are pretty prevalent in distribution power systems with intermittent energy resources like EVs present. It is proven to converge which is shown using the Lyapunov direct method and the analytical results were tested on IEEE 123 bus test system. The results obtained are also included for the illustration of the proposed algorithm.

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A Appendix

Consider the following properties of the convex objective function $f_i(y_i)$:

- The functions $f_i(y_i)$ is convex and differentiable. In particular, the gradient of a convex function is a global under-estimator as:

$$f_i(y_i) - f_i(y_i^*) \geq \nabla_{y_i^*} f_i(y_i^*)(\tilde{y}_i). \tag{28}$$

and

$$f_i(y_i^*) - f_i(y_i) \geq \nabla_{y_i}^T f_i(y_i)(-\tilde{y}_i) \tag{29}$$

Adding the above two inequalities yields

$$[-\nabla_{y_i}^T f_i(y_i) + \nabla_{y_i^*}^T f_i(y_i^*)]^T \tilde{y}_i \leq 0. \tag{30}$$

- The gradient of $f_i(y_i)$ (denoted by $\nabla_{y_i} f(y_i)$) is Lipschitz, i.e., $||\nabla_{y_i} f(y_i) - \nabla_{y_i^*} f(y_i^*)|| \leq l_i ||y_i - y_i^*||$, where l_i is Lipschitz constant.

From (30), we get $[\nabla_{y_i}^T f_i(y_i) - \nabla_{y_i^*}^T f_i(y_i^*)]^T \tilde{y}_i$ which is positive definite with respect to $\tilde{y}$.

Lemma 1: If the gradients denoted by $\nabla_{y_i} f_i(y_i)$ are locally Lipschitz, the sum

$$-\sum_l k_i ||\epsilon_{il}||^2 - \alpha_i \left[\sum_l \epsilon_{il}\right]^T \nabla_{y_i} f_i(y_i) - \alpha \tilde{y}_i^T \nabla_{y_i} f_i(y_i) \tag{31}$$

is negative definite with respect to ϵ_{il} and $\tilde{y}_i$ for small stepsize α_i and for all gain $k_i > 0$ above a certain threshold.

Consider the Lyapunov function

$$\begin{aligned} V =& \frac{1}{2} \sum_{i=1}^{N} \Bigg\{ 2V_i^{'}(x_i, \omega_i) + ||\tilde{\omega}_i||^2 + \frac{1}{w_i} ||\tilde{\mu}_i||^2 + \sum_{j \in \mathcal{N}_i} \left[\frac{1}{\alpha_i} ||\tilde{z}_{ij}||^2 + ||\tilde{\lambda}_{ij}||^2 \right. \\ &+ \left. d_{ij} \int_{t-\delta}^{t} ||\tilde{z}_{ij}(\tau)||^2 d\tau \right] \Bigg\} \end{aligned}$$

It follows from (14) that

$$\begin{aligned} \dot{V} \leq& \sum_{i=1}^{N} \Bigg\{ -\sum_l k_i ||\tilde{\omega}_i - C_{il}\tilde{x}_i||^2 + \left[\sum_l (\tilde{\omega}_i - C_{il}\tilde{x}_i)\right]^T \dot{\tilde{\omega}} + \tilde{y}_i^T \dot{\tilde{\omega}}_i + \tilde{\mu}_i^T \dot{\tilde{\mu}}_i \\ &+ \sum_{j \in \mathcal{N}_i} \left[\tilde{z}_{ij}^T \dot{\tilde{z}}_{ij} + \tilde{\lambda}_{ij}^T \dot{\tilde{\lambda}}_{ij} + \frac{1}{2} d_{ij} \left[||\tilde{z}_{ij}||^2 - ||\tilde{z}_{ij}^-||^2 \right] \right] \Bigg\} \\ =& \sum_{i=1}^{N} \Bigg\{ -\sum_l k_i ||\omega_i - C_{il}\tilde{x}_i||^2 - \alpha_i \left[\sum_l \omega_i - C_{il}\tilde{x}_i\right]^T \nabla_{y_i} f_i(y_i) \\ &- \alpha \tilde{y}_i^T \nabla_{y_i} f_i(y_i) + \sum_{j \in \mathcal{N}_i} \Bigg[\tilde{\mu}_i A_{ji} \tilde{z}_{ij} + \tilde{\xi}_i^T \Big[-d_{ij} \tilde{\lambda}_{ij} - d_{ij}(\tilde{\xi}_i - \tilde{z}_{ij}^-) \Big] \\ &+ \tilde{z}_{ij}^T \Big[d_{ij} \tilde{\lambda}_{ij} + d_{ij}(\tilde{\xi}_i - \tilde{z}_{ij}) - A_{ji}^T \tilde{\mu}_i \Big] + d_{ij} \tilde{\lambda}_{ij}^T \Big[\tilde{\xi}_i - \tilde{z}_{ij} \Big] \\ &+ \frac{1}{2} d_{ij} ||\tilde{z}_{ij}||^2 - \frac{1}{2} d_{ij} ||\tilde{z}_{ij}^-||^2 \Bigg] \Bigg\} \end{aligned}$$

$$= \sum_{i=1}^{N} \Bigg\{ \underbrace{-\sum_{l} k_i ||\omega_i - C_{il}\tilde{x}_i||^2 - \alpha_i \left[\sum_{l} \omega_i - C_{il}\tilde{x}_i\right]^T \nabla_{y_i} f_i(y_i) - \alpha \tilde{y}_i^T \nabla_{y_i} f_i(y_i)}_{\text{n.d. according to (31)}}$$

$$- \frac{1}{2} d_{ij} ||\tilde{\xi}_i - \tilde{z}_{ij}||^2 - \frac{1}{2} d_{ij} ||\tilde{\xi}_i - \tilde{z}_{ij}^{-}||^2$$

which is negative definite with respect to $(\tilde{\xi}_i - \tilde{z}_{ij})$ as well as $(\tilde{\xi}_i - \tilde{z}_{ij}^{-})$. This concludes the proof. ■