A Novel State-Constrained Primary Control for Grid-Forming Inverters

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Abstract—The problem of operating an AC microgrid with purely inverter-based generating units is considered, and a novel vector control for the constrained grid-forming (GFM) inverters is proposed. The design aims to achieve the primary control objectives of the GFM inverters: voltage establishment, synchronization, and voltage/frequency regulation. It is novel that the proposed control also ensures all operational values constrained both real-time and for all the time except for low voltage during the period of grid forming. The constraints include frequency, output AC voltage, input DC power, and active/reactive power injections to the grid. The proposed control has three parts: the nominal frequency control to track the reference signals of frequency and angle, the nominal voltage control to track the voltage magnitude, and a Lyapunov-based state-constrained control to ensure that the aforementioned constraints are dynamically satisfied while incorporating intermittent renewables. The proposed control can follow the frequency, angle, and voltage references that come from an appropriately designed secondary control. In this paper, outcomes of a detailed simulation are presented to demonstrate that the above design objectives are met and that the overall performance is superior due to its unique ability of simultaneously controlling the whole vectors of the inverter state and output.

Index Terms—vector control, grid-forming inverter, feedback linearization, frequency and voltage regulation, constrained control, microgrid.

I. INTRODUCTION

FUTURE electric power grids are expected to be capable of operating on up to 100% renewable distributed energy resources (DERs), which seem to be the potential substitution of fossil energy. Many economic, technical, and political challenges and obstacles have to be conquered before the power industry evolves and upgrades to its next generation. Among those, maintaining the resilience and stability of voltage and frequency is a fundamental question yet to answer. Traditional power systems usually have a sophisticated operational regime based on the synchronous generators’ (SMs) multilevel control, which has been developed and improved for over a hundred years. While most of the DERs are based on inverters so that they are very different from the synchronous generators. The small capacity of each DER, the large number of generation units, and the intermittent outputs make it impossible to control the future grids in the same way as before. The grid-forming (GFM) inverter control has been under the spotlight as a potential candidate to fill in this role in the future.

The main direction of GFM design is the droop-based control and its variations [1], [2]. For instance, the virtual synchronous machine control [3]–[6] has been utilized as the primary control for GFM inverters to mimic the inertia of SMs. This research showed significant improvement in the transient response of the inverter control during disturbances. However, the droop-based design’s well-known flaw is the deviation from reference value. To tackle this problem and clarify the application functionalities in different time scale, a hierarchical implementation of primary control, secondary control, and tertiary control is developed in [7], [8]. Another question about the droop methods is how to properly set the control parameters for millions or more DERs in a large-scale power system. The droop design linearly generates the references of frequency and voltage amplitude with real and reactive power output, respectively. The control gains and references used for one inverter might conflict with others, which is still an unsolved challenge. Moreover, in the droop-based approaches, the voltage and current tracking control are traditionally included as the inner loops. They are considered part of the inverter model in many papers [9], though it is not necessary.

In contrast, another category of GFM design does not apply the above-mentioned inner loops, and is called the direct nonlinear methods. The virtual oscillator controller (VOC)-based design [10], [11] is an emerging and promising direction of the nonlinear approach and followed by many. The VOC design is conducted directly on the abc-frame, and the synchronization is assured for connected VOC inverters based on the oscillator synthesis theory [11]. But there are some ongoing issues with VOC from the application perspective, such as the harmonic current caused both by the oscillator and by the AC network. And neither this method has been implemented nor its effectiveness has been justified in more extensive systems with disperse DERs. Hence many researchers are still working to improve the VOC method [12], [13].

Inspired by all the exploring work on GFM approaches, we develop a novel nonlinear vector control that enables the inverter’s outputs to track their set-points from the system level and dynamically meet the operational constraints. The proposed design focuses on the objectives of the GFM inverters’ primary control, i.e., voltage establishment, synchronization, and voltage/frequency regulation; while the references are generated from a secondary control based on a distributed subgradient method [14]–[19] aiming to reallocate the power generation between DERs. In this way, the overall control
structure is completed and the performance has been tested in the following simulation.

Compared to the existing work, the new method is a systematic design, which is based on the complete models of the inverter, the output filter, the phase-sequence transformation, and the AC network [20]; hence it is applicable to any scale of smart grids (or distribution networks) regardless the connection and capacity of the DERs. Moreover, the operational constraints are met for all the time (except the establishing period).

The remainder of this paper is organized as follows. Section II describes the dynamic model for GFM inverters and formulates the control design problem. In Section III we present the nominal frequency/voltage control, and the state/input-constrained control. A step-by-step simulation is performed and all the functions designed are verified in Section IV. The conclusions are drawn in Section V.

II. PROBLEM FORMULATION

The control design is developed based on an inverter-based AC microgrid model, which has three parts: the inverter and its output filters, the abc-dq0 transformation, and the AC network, as shown in Fig. 1. The input/output signals and the primary objectives of the control block will be analyzed in this section.

A. Model

1) abc-dq0 Transformation: The graphic demonstration of the average model of the kth inverter and its output filters is shown in Fig. 1 and Fig. 2. As seen in both figures, the frequency and angle of the inverter are denoted by \( \omega_k \) and \( \delta_k \), respectively; and we have

\[
\dot{\delta}_k = \omega_k. \tag{1}
\]

The voltages and currents on different parts of the LCL filter are denoted by \( v_{sk}, i_{sk}, v_{ok}, i_{ok} \); the voltage on the PCC bus is denoted by \( u_{sk} \), and the control is denoted by \( u_k \). The variables \( v_{sk}, i_{sk}, v_{ok}, i_{ok}, v_k, \) and \( u_k \) are all vectors of either abc or dq0-frame values depending on the context. The \( T \) matrix between abc and dq0-frame is defined with angle \( \delta_k \)

\[
T(\delta_k) = \frac{2}{3} \begin{bmatrix}
\cos(\delta_k) & \cos(\delta_k - \frac{2\pi}{3}) & \cos(\delta_k + \frac{2\pi}{3}) \\
-\sin(\delta_k) & -\sin(\delta_k - \frac{2\pi}{3}) & -\sin(\delta_k + \frac{2\pi}{3})
\end{bmatrix},
\]

and the mapping is as follows,

\[
x_{dq0} = T(\delta_k)x_{abc} \quad \text{and} \quad x_{abc} = T^{-1}(\delta_k)x_{dq0}.
\]

For simplicity of expressions, the abc or dq0-frame denotations for all these vectors are not specified in the rest of the paper. In the control design part of this paper, all these vectors are specified as two-dimensional vectors with only dq-sequence values, and the zero-sequence will be always zero when mapping them from dq0 to abc-frame.

2) Vector Control Model of the GFM Inverters: The output frequency is assumed to be equal to the input frequency of the average inverter model, which means

\[
\omega_k = \omega_{ik}. \tag{2}
\]

The dynamic model of the kth GFM inverter and its output filters [21] is written as follows:

\[
\frac{di_{ok}}{dt} = A_{11,k}i_{ok} + A_{12,k}v_{ok} + D_{1,k}v_{bk} \tag{3}
\]

\[
\frac{dv_{ok}}{dt} = A_{21,k}i_{ok} + A_{22,k}v_{ok} + A_{23,k}i_{sk} \tag{4}
\]

\[
\frac{di_{sk}}{dt} = A_{32,k}v_{ok} + A_{33,k}i_{sk} + B_{3,k}u_k, \tag{5}
\]

where

\[
A_{11,k} = \begin{bmatrix}
-R_{ck} & \omega_0 & -R_{ck} \\
-\omega_0 & -R_{ck} & \omega_0 \\
R_{ck} & \omega_0 & -R_{ck}
\end{bmatrix}, \quad A_{12,k} = \begin{bmatrix}
L_{ck} & 0 & 0 \\
0 & L_{ck} & 0 \\
0 & 0 & L_{ck}
\end{bmatrix},
\]

\[
A_{21,k} = \begin{bmatrix}
-\frac{1}{C_k} & 0 & 0 \\
0 & -\frac{1}{C_k} & 0 \\
0 & 0 & -\frac{1}{C_k}
\end{bmatrix}, \quad A_{22,k} = \begin{bmatrix}
0 & 0 & 0 \\
0 & \omega_0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
A_{32,k} = \begin{bmatrix}
-\frac{1}{L_{fk}} & 0 & 0 \\
0 & -\frac{1}{L_{fk}} & 0 \\
0 & 0 & -\frac{1}{L_{fk}}
\end{bmatrix}, \quad A_{33,k} = \begin{bmatrix}
R_{fk} & \omega_0 & -R_{fk} \\
\omega_0 & -R_{fk} & \omega_0 \\
-R_{fk} & \omega_0 & -R_{fk}
\end{bmatrix},
\]

\[
D_{1,k} = -A_{12,k}, \quad A_{23,k} = -A_{21,k}, \quad B_{3,k} = -\frac{v_{dc,k}}{2}A_{32,k},
\]

\( \omega_0 \) is the nominal frequency, \( v_{dc,k} \) is the DC-side voltage, and all other parameters of different parts of the LCL filter are defined as shown in Fig. 1.

The output voltage magnitude, active and reactive powers of each inverter are defined by the above vectors as follows:

\[
V_k = \|v_{ok}\|, \tag{6}
\]

\[
P_k = \frac{3}{2}v_{ok}^\top i_{ok}, \quad Q_k = \frac{3}{2}v_{ok}^\top H_q v_{ok}, \tag{7}
\]
where \( H_q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \).

### B. Constraints and Control Objectives

The operation of GFM inverters is generally constrained by the following two sets of conditions: the regulation requirements on voltages and frequency, and the output power limits of DERs. The regulation constraints are summarized as below: for \( k = 1, \cdots, N_q \) and for all time (except for the initial period of grid forming),

\[
\omega_k \in [\omega_i, \omega_f] \quad (8) \\
V_k \in [V_0 - \Delta_V, V_0 + \Delta_V], \quad (9) \\
|v_{q,k}| \leq \epsilon_v \quad (10)
\]

where \( \epsilon_v > 0 \) is a design parameter, \( V_0 \) is the nominal voltage, \( \omega = \omega_0 - \Delta \omega \), \( \omega = \omega_0 + \Delta \omega \), \( \Delta \omega, \Delta V > 0 \) is the maximum tolerance of frequency/voltage derivations (e.g. the typical value for voltage deviation is about 5% – 10%). And the power constraints are the limits on each DG,

\[
P_k(t) \leq P_k \leq \bar{P}_k(t) \\
-Q_k(t) \leq Q_k \leq \bar{Q}_k(t)
\]

where \( S_k \) is the apparent power capacity of inverter \( k \), \( \bar{Q}_k(t) = \sqrt{S_k^2 - \bar{P}_k^2(t)} \), \( \bar{P}_k(t) \) is the upper limit of the real power injection. In other words, \( \bar{P}_k(t) \) represents the real power limit from its source side, which may be time varying, e.g. PV and other DERs are intermittent. Typically, \( \bar{P}_k(t) = 0 \) for DGs, but it may assume negative values for storage devices.

Assuming all the state vectors in (3) to (5) can be measured, \( u_k \) is the control to be designed. The voltage, frequency, and angle references are denoted as \( V_{\text{ref}}^k, \omega_{\text{ref}}^k, \) and \( \delta_{\text{ref}}^k \), which will be the design focus of secondary control (refer to [14], [21]).

In summary, the primary control objectives are:

- Frequency control \( \omega_{\text{ck}} \) is designed to make \( \omega_k \) and \( \delta_k \) track \( \omega_{\text{ref}}^k \) and \( \delta_{\text{ref}}^k \), respectively, while satisfying constraint (8);
- Given output voltage reference \( V_{\text{ref}}^k \) from the secondary control, find \( u_{\text{ref}}^k \) which satisfies constraint (10);
- A nominal tracking control \( u_k^* \) is designed for \( u_k \) to follow voltage reference \( u_{\text{ref}}^k \);
- Based on \( u_k^* \), control \( u_k \) is designed to meet all the input/output/state constraints in (9), (10), (11) and (12).

### III. CONTROL DESIGN

The proposed primary control, as illustrated in Fig. 3, has the following three components: 1) the nominal frequency control that tracks the reference signals of the angle and frequency; 2) the nominal voltage control which tracks the voltage reference; 3) an input-state constrained control which is to modify the nominal control in order to ensure that the constraints on the input and the state are satisfied dynamically.

\[\text{Fig. 3. The structure of the primary control on GFM inverters}\]

#### A. Nominal Frequency Control

The frequency control, \( \omega_{\text{ck}} \), aims to make \( \omega_k \) and \( \delta_{\text{ck},o} \) track \( \omega_{\text{ref}}^k \) and \( \delta_{\text{ref}}^k \), respectively, while satisfying constraint (8). The design is given by the following lemma.

**Lemma 1.** Consider frequency control \( \omega_{\text{ck}} \) in the form of

\[
\omega_{\text{ck}} = \text{SAT}_{[\omega_i, \omega_f]}(\omega_k), \\
\omega_{\text{ck}} = \omega_0 - \int_{t_{k,0}}^t [2\gamma_{\text{w}}(\omega_k - \omega_{\text{ref}}^k) + \gamma_{\text{w}}^2(\delta_k - \delta_{\text{ref}}^k)] dt \quad (13)
\]

where \( \gamma_{\text{w}} \) is a small positive gain, \( \omega_{\text{ref}}^k \) and \( \delta_{\text{ref}}^k \) are given by the secondary-level algorithm, and \( \text{SAT}_{[\omega_i, \omega_f]}(\cdot) \) is the saturation function with the lower/upper limit of \( \omega/\omega_0 \). Then, under frequency control (13), frequency \( \omega_k \) and phasor \( \delta_k \) converge to their reference values \( \omega_{\text{ref}}^k \) and \( \delta_{\text{ref}}^k \), respectively.

**Proof of lemma 1:** It follows from (1), (2), and (13) that

\[
\frac{d}{dt}(\omega_k) = d(\omega_k) = -2\gamma_{\text{w}}(\omega_k - \omega_{\text{ref}}^k) - \gamma_{\text{w}}^2(\delta_k - \delta_{\text{ref}}^k) \leq 0
\]

which shows critically-damped asymptotic convergence provided that \( \omega_k \) and \( \delta_k \) converge to \( \omega_{\text{ref}}^k \) and \( \delta_{\text{ref}}^k \), respectively, and \( \omega_{\text{ref}}^k \) converges to \( \omega_0 \).

#### B. Nominal Voltage Control

The nominal voltage control at the primary level, denoted by \( u_k \), is designed to make \( dq \)-frame vectors \( v_{\text{ok}} \) to track its reference \( v_{\text{ref}}^{\text{ok}} \). As illustrated in Fig. 4, and \( v_{\text{ref}}^{\text{ok}} \) should be on \( dq \)-axis of the reference \( dq \)-frame with angle \( \delta_{\text{ref}}^k \), such that

\[
v_{\text{ref}}^{\text{ok}} = [V_{\text{ref}}^k, 0]^T.
\]

Given \( v_{\text{ok}} \) and \( v_{\text{bk}} \) (to be measured), the reference vectors of \( i_{\text{ok}}, i_{\text{bk}} \), and \( u_k \) can be solved from the steady-state forms of (3) to (5), i.e.,

\[
i_{\text{ok}}^r = A_{11,k}^{-1}[-A_{12,k}v_{\text{ok}}^{\text{ref}} - D_{1,k}v_{\text{bk}}], \\
i_{\text{bk}}^r = A_{21,k}^{-1}[-A_{22,k}v_{\text{ok}}^{\text{ref}} - A_{23,k}v_{\text{bk}}], \\
u_k^r = A_{31,k}^{-1}[-A_{32,k}v_{\text{ok}}^{\text{ref}} - A_{33,k}v_{\text{bk}}],
\]

The feedback linearization technique [22] is used for the control design, and the result is summarized as the following lemma.
Lemma 2. Given the reference vector \( v^r_{ok} \) on the d-axis of the reference dq-frame, the control \( u_{ck} \) can make \( v_{ok} \) track it under the following law:

\[
u_{ck} = -B^{-1}_{3,k}[A_{33,k}i_{sk} + A_{32,k}v_{ok} + \gamma_{ik}(i_{sk} - i^r_{sk}) - di^r_{sk}/dt],
\]

where \( \gamma_{ik}, \gamma_{ik} > 0 \) are control gains,

\[
i^r_{sk} = -A^{-1}_{23,k}[A_{21,k}i_{ok} + A_{22,k}v_{ok} + \gamma_{ik}(v_{ok} - v^r_{ok})],
\]

and

\[
di^r_{sk}/dt = -A^{-1}_{23,k}[(A_{21,k}A_{11,k} + A_{22,k}A_{21,k} + \gamma_{ik}A_{21,k})i_{ok} + (A_{21,k}A_{12,k} + A^2_{22,k} + \gamma_{ik}A_{22,k})v_{ok} + A_{21,k}D_{1,k}v_{bk} + (A_{22,k}A_{23,k} + \gamma_{ik}A_{23,k})i_{sk}].
\]

The nominal control can make the GFM inverters track their corresponding references, but it may not satisfy the real-time operational constraints. To this end, the constrained design is developed based on the comparison lemma [21], [24], [25], which is restated as follows. If a first order scalar ODE \( \dot{x} = f(x) \) with the initial \( x(t_0) \) has the solution \( x = h(t) \), then the differential inequality \( \dot{x} \leq f(x) \) with \( x(t_0) \) has the solution of \( x \leq h(t) \). For example, consider a first-order system: \( \dot{x} = u \), which has the feasible set \( \Omega \triangleq [x^* - \sigma, x^* + \sigma] \). Choose \( u \) such that \(-x + x^* - \sigma \leq u \leq -x + x^* + \sigma \), then \( x \to \Omega \), as shown in Fig. 5.

C. Constrained Control Design

It follows the system dynamic equations that \( V_k, P_k \) and \( Q_k \) are all with relative degree two and their constrained control can fit the high-order form of the comparison lemma, which is explained as follows. Consider the following differential inequality.

\[
\prod_{j=1}^{\mu_i} (s + \gamma_{ij}) \xi_i(t) \leq 0, \tag{24}
\]

where \( \gamma_{ij} \) are constants. The solution \( \xi_i(t) \) has the property that the following inequalities hold whenever \( \mu_i > 1 \):

\[
\prod_{j=1}^{k} (s + \gamma_{ij}) \xi_i(t)_{t=0} \leq 0, \forall k \in \{1, \ldots, \mu_i - 1\},
\]

if \( \xi_i(t) \neq 0 \), then \( \xi_i(t) \neq 0 \) becomes true exponentially under inequality (24). Since \( \mu_i = 2 \) for \( V_k, P_k \) and \( Q_k \), their constrained control design can be performed and inequality projections on \( u_k \) can be derived accordingly.

\[
M_i \leq G_i u_k \leq E_i, \tag{25}
\]

where the index \( i \) represents the number of constraints considered, matrix \( G_i \) and vectors \( M_i, E_i \) represent the results of constrained design when performing the above design on system dynamic equations, (3), (4), (5), (6), and (7) (See [21] for a representative result).

IV. Simulation and Experimental Results

A three-inverter microgrid is built for the tests, as shown in Fig. 6. Without loss of generosity of AC grid, both R and RL-type loads are used in the simulation, and the parameters are provided in table I.
1) **One-Inverter Scenario**: In this case, only one inverter (DER #1) is turned on to test the primary voltage control and the constrained control.

**Nominal Voltage Control**: The frequency is set to be $\omega_1 = \omega_0$, and the voltage reference is set to be $V_{1r} = V_0$. The simulation result of the nominal voltage control (18) is shown in the following figure. The output frequency is the nominal value, i.e. $\omega_1 = \omega_0$; the voltage of this system is successfully established through (18); and certain amount of power is generated from DER #1 accordingly. Note that the frequency measured from PLL has some dynamics inherited from PLL process.

**Constrained Control**: With the same settings, the real power is limited to $P_1 = 6kW$ at the maxim, and the constrained control has been tested and the result is shown in the Fig. 8. As in the figures below, Once the real power has been in its feasible region, the control input $u_k$ will be constrained such that $P_k$ will never go beyond its limits. In this case, $u_d$ is constrained at $\pi_d$, the power $P_k$ is constrained at $P$, and the voltage $V_{1,a}$ is settled at a lower value (other than $V_0$).

A separate case with the reactive power constrained is tested by setting $Q_k = 0.6kVA$, and the results is shown in figure 9: $u_k$ is constrained when $Q_k$ approaches to its upper limit $Q$, and the voltage $V_{1,a}$ is a lower value (other than $V_0$) in the steady state.

2) **Two-Inverter Scenario**: Since the angle is a relative variable, the nominal frequency control is tested in the two-inverter case, which has two DERs (#1 and #2) turned on.

**Primary Frequency/Angle Control**: In this case, the frequency and angle tracking functions of the primary frequency control (13) are testified. The angle reference of DER #2 is set to be $\delta_{2r} = -0.006$ at time point $0.8s$, the system response is shown in figure 10. The result shows that the inverter angle asymptotically converge to the reference through the control law designed. It is worth to note that angle difference between inverters has significant impact on the power flow, which shows the potential of angle and voltage-based secondary control.

3) **Three-Inverter Scenario**: The overall performance is tested by applying the proposed control with a subgradient cooperative control [21] in the following three-inverter case.

**Constrained Control**: As seen in the results of figure 11a, the voltage of DER #1 is constrained when the power supply is insufficient in the system. The power does not exceed its limit.
as shown in figure 11b. The secondary level control drives both the real and reactive power (utilization ratios) reach consensus, separately.

V. CONCLUSION
A novel primary control is developed for constrained grid-forming (GFM) inverters. Three distinctive components are delivered: the nominal frequency control, the nominal voltage control, and the constrained control. The nonlinear nominal control can asymptotically track the reference signals of frequency, angle, and voltage magnitude. The constrained control design imposed on the nominal control maintains all the operating states within their limits. The proposed control is scrutinized in a test system of microgrid through step-by-step simulation. The result shows that the proposed control can smoothly establish the AC system while enabling all the constraints on inputs and states. Further work will be carried on focusing on integrating the novel GFM design with the legacy inverters with the traditional controls.

REFERENCES