Oscillatory Power Control and Consensus in Unbalanced Networks Using Grid Forming Inverters

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***Abstract*—In this paper, a state-space nonlinear control previously designed for balanced operation of grid-forming (GFM) inverters is extended to unbalanced operations. Unbalanced voltages and currents are analyzed, and their time-varying direct, quadrature, and zero components are parameterized in terms of six constant parameters. These parameters provide primary control design choices for not only an inverter's terminal voltage but also its injections of both real and oscillatory power. It is shown that the proposed GFM control can precisely achieve a desired balanced terminal voltage under unbalanced loads. It is also shown that, by allowing the terminal voltage to be slightly unbalanced, oscillatory power can be adaptively minimized. Finally, it is illustrated that oscillatory power can also be shared among multiple GFM inverters. Case studies and their simulations demonstrate that the proposed GFM control is effective in meeting the aforementioned objectives.**

Index terms: Grid-forming control, unbalanced networks, oscillatory power, voltage regulation, power sharing.

I. Introduction

Renewable energy and inverter-based resources will play a paramount role in zero-carbon power and energy systems. Grid-forming (GFM) inverters are critical to form and maintain grid voltages, especially in black start and/or microgrid operation. To ensure reliability and resilience, GFM inverters must also operate safely and effectively under all conditions, in particular, unbalanced conditions.

In both power system analysis and control, unbalanced 3 phase quantities are typically expressed in terms of a synchronous rotating frame (dqz frame). Under unbalanced conditions, dqz quantities by nature are oscillatory. This means that dqz quantities cannot be regulated well by PI controllers without the use of notch filters. But, those filters remove the 2ω oscillation of the dqz quantities and introduce delays, and hence the resulting control is approximately for the DC components only. To obtain stationary dqz quantities, additional Fortescue's transform [1], [2] is required to transform the abc quantities to $+-0$ quantities and then to dqz. This increases the number of controllers required and consequently the computational burden.

In [3], a grid forming scheme under unbalanced conditions is considered, and an $\alpha\beta$-based PI control is used by skipping the transform from abc to $+-0$ to reduce the computational burden. In addition, the 0 component is ignored in [3], the resulting control fails to maintain balanced voltages as proposed, and it also requires the use of an additional grounding transformer.

In this paper, a nonlinear state-space GFM control is synthesized by using the recursive design process and by extending the result in [4]. The proposed control uses the aforementioned dqz quantities and precisely controls 3-phase voltages in unbalanced systems. Different from [3], the proposed GFM control can achieve balanced terminal voltages for any unbalanced system. Another key contribution of this paper is oscillatory power control and sharing. A review of different reactive power definitions under unbalanced conditions is presented in [5] and [6]. Although there may never be a consensus on reactive power in general, it is straightforward to quantify oscillatory power, and control it. Two sets of case studies demonstrated the effectiveness of the proposed method and controls.

The rest of the paper is organized as follows. Section II describes the dqz quantities for unbalanced voltages or currents and a dqz model of GFM inverter and its LCL filter. Section III provides the instantaneous, real, and oscillatory power of unbalanced operation. And, Section IV and V include GFM control, oscillatory power control, power sharing, and the corresponding case-study results.

II. Model Description

A. Park Transform

The Park transformation converts a time-domain vector of three-phase abc components to the vector of direct, quadrature, and zero (dqz) components in the rotational reference frame, expressed as,

$$x_{dqz} = T_{dqz} x_{abc}$$

where

$$T_{dqz}(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

$\theta(t)$ is the phase angle with $\theta(t) = \int_0^t \omega(\tau)d\tau + \theta_0$, and θ_0 is the initial orientation angle between the dqz frame and the synchronous frame. Its inverse transformation is given by

$$T_{dqz}^{-1}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 1 \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & 1 \\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & 1 \end{bmatrix}$$

In what follows, the transformation matrix with $\theta_0 = 90^0$ is used. In other words, initial orientation of the dqz-frame is such that the q-axis is aligned with the a-axis.

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B. Unbalanced dqz Quantities

Consider the following set of three-phase unbalanced vectors:

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} X_a \sin(\omega t + \phi_a) \\ X_b \sin(\omega t - 2\pi/3 + \phi_b) \\ X_c \sin(\omega t + 2\pi/3 + \phi_c) \end{bmatrix}$$

Applying the dqz transformation, $T_{dqz}(\theta)$, to the above vector and using trigonometric identities, we have

$$\begin{aligned} x_{dqz} &= \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} \\ &= \begin{bmatrix} X_1 & 0 & 0 & -X_3 & X_4 \\ X_2 & 0 & 0 & X_4 & X_3 \\ 0 & X_5 & X_6 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \cos(\omega t) \\ \sin(\omega t) \\ \cos(2\omega t) \\ \sin(2\omega t) \end{bmatrix}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} X_1 &= \frac{1}{3}[X_a \cos\phi_a + X_b \cos\phi_b + X_c \cos\phi_c], \\ X_2 &= \frac{1}{3}[X_a \sin\phi_a + X_b \sin\phi_b + X_c \sin\phi_c], \\ X_3 &= \frac{1}{3}\left[X_a \cos\phi_a + X_b \cos(A) + X_c \cos(B)\right], \\ X_4 &= \frac{1}{3}\left[X_a \sin\phi_a + X_b \sin(A) + X_c \sin(B)\right], \\ X_5 &= \frac{1}{3}\left[X_a \sin\phi_a + X_b \sin(C) + X_c \sin(D)\right], \\ X_6 &= \frac{1}{3}\left[X_a \cos\phi_a + X_b \cos(C) + X_c \cos(D)\right], \\ A &= \phi_b - \frac{4\pi}{3}, \quad B = \phi_c + \frac{4\pi}{3}, \\ C &= \phi_b - \frac{2\pi}{3}, \quad D = \phi_c + \frac{2\pi}{3}. \end{aligned}$$

The above is applicable for both currents and voltages with imbalance in magnitude and/or phase. Note that, in the dqz quantities, X_1 through X_6 are constants during steady-state.

C. Real-Time Computation

The quantities X_1 up to X_6 are defined in terms of peak value and angles of three phase sinusoidal functions $f_i(t)$. These peaks and angles can be obtained by real-time convolution of function $f_i(t)$ with the standard sinusoidal signals. It follows from Fourier transform that if

$$f_i(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

where $T = 2\pi/\omega$, ω is the fundamental frequency, and

$$\begin{aligned} a_n &= \frac{2}{T}\int_{t-T}^{t} f_i(\tau)\cos(n\omega\tau)d\tau \\ b_n &= \frac{2}{T}\int_{t-T}^{t} f_i(\tau)\sin(n\omega\tau)d\tau. \end{aligned}$$

then

$$\begin{aligned} |f_i(t)|_n &= \sqrt{a_n^2 + b_n^2} \\ \angle[f_i(t)]_n &= \arctan\left(\frac{a_n}{b_n}\right), \end{aligned}$$

where $|f_i(t)|_1$ and $\angle[f(t)]_1$ are the required magnitudes and angles.

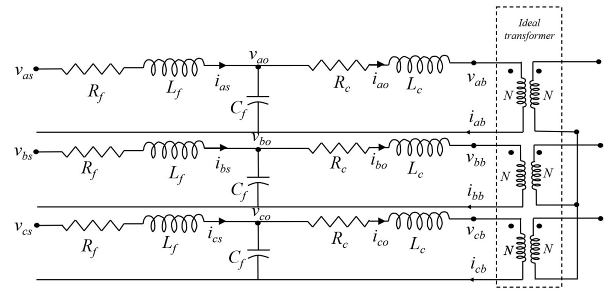

Fig. 1. LCL filter diagram with $Y-Y$ isolation transformer

D. GFM Inverter and Its LCL Filter

In this paper, GFM inverters are represented by the following average model in the dqz frame:

$$v_s = \frac{2}{V_{dc}}u,$$

where V_{dc} is the DC-link voltage, v_s is the output voltage after the switches, and u is the control input in dqz. The inverter's LCL filter is shown in figure 1. It follows from KCL and KVL that the dynamic state equations in the abc domain are:

$$L_c\frac{di_o}{dt} = -R_c i_o + v_o - v_b, \quad (2)$$

$$C\frac{dv_o}{dt} = i_s - i_o, \quad (3)$$

$$L_f\frac{di_s}{dt} = -R_f i_s + v_s - v_o, \quad (4)$$

where

$$v_s = \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix}, \quad v_o = \begin{bmatrix} v_{oa} \\ v_{ob} \\ v_{oc} \end{bmatrix}, \quad v_b = \begin{bmatrix} v_{ba} \\ v_{bb} \\ v_{bc} \end{bmatrix},$$

$$i_s = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}, \quad i_o = \begin{bmatrix} i_{oa} \\ i_{ob} \\ i_{oc} \end{bmatrix}.$$

Applying the dqz transformation yields the following equations in the dqz domain:

$$\dot{i}_o = A_{11}i_o + A_{12}v_o + D_1 v_b \quad (5)$$

$$\dot{v}_o = A_{21}i_o + A_{22}v_o + A_{23}i_s \quad (6)$$

$$\dot{i}_s = A_{31}i_s + A_{32}v_o + B_3 u \quad (7)$$

where

$$W = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_{11} = \left[W - \frac{R_c}{L_c}I_{3\times3}\right], \quad A_{12} = \frac{1}{L_c}I_{3\times3}, \quad D_1 = -\frac{1}{L_c}I_{3\times3},$$

$$A_{22} = W, \quad A_{21} = -\frac{1}{C} I_{3\times3}, \quad A_{23} = \frac{1}{C} I_{3\times3},$$

$$A_{31} = \left[W - \frac{R_f}{L_f} I_{3\times3} \right], \quad A_{32} = -\frac{1}{L_f} I_{3\times3},$$

$$B_3 = \frac{V_{dc}}{2}\frac{1}{L_f} I_{3\times3}, \quad v_s = \begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{s0} \end{bmatrix}, \quad v_o = \begin{bmatrix} v_{od} \\ v_{oq} \\ v_{o0} \end{bmatrix},$$

$$v_b = \begin{bmatrix} v_{bd} \\ v_{bq} \\ v_{b0} \end{bmatrix}, \quad i_s = \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{s0} \end{bmatrix}, \quad i_o = \begin{bmatrix} i_{od} \\ i_{oq} \\ i_{o0} \end{bmatrix}.$$

The coupling between the d- and q-axis arises due to the fact that

$$\frac{dT_{dqz}^{-1}}{dt} = -T_{dqz}^{-1} W.$$

III. Instantaneous, Real, and Oscillatory Power under Unbalanced Conditions

It follows from definition that instantaneous power in the dqz domain is

$$\begin{aligned} p(t) &= \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \\ &= \begin{bmatrix} v_d & v_q & v_0 \end{bmatrix} (T_{dqz}^{-1})^T T_{dqz}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \\ &\triangleq v_{dqz}^T H_p i_{dqz}, \end{aligned}$$

where

$$H_p = \frac{3}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Expressing the voltage and current expressions in terms of (1) and multiplying the above expression yields

$$\begin{aligned} p(t) &= \frac{3}{2}\left[P + O_1 \cos 2\omega t + O_2 \sin 2\omega t\right] \\ &\triangleq P - O \sin(2\omega t + \phi'), \end{aligned}$$

where P is the *active power* defined by

$$P = \frac{3}{2}(V_1 I_1 + V_2 I_2 - V_3 I_3 + V_4 I_4 + I_5 V_5 + I_6 V_6), \quad (8)$$

and O is the so-called *oscillatory power* defined by

$$O = \sqrt{O_1^2 + O_2^2},$$

where

$$\begin{aligned} O_1 &= \frac{3}{2}(V_1 I_3 - V_3 I_1 + V_2 I_4 + I_2 V_4 + I_5 V_5 - I_6 V_6), \quad (9) \\ O_2 &= \frac{3}{2}(V_1 I_4 + V_4 I_1 + V_3 I_2 - V_2 I_3 + V_6 I_5 + V_5 I_6), \quad (10) \end{aligned}$$

and

$$\phi' = \text{atan}\frac{O_1}{O_2}.$$

The above definition of oscillatory power in the dqz quantities allow us to investigate how to control all the time-varying power components under the unbalanced operation. In particular, the oscillatory power can be controlled in terms of time-invariant dqz components as defined in (1), that is, V_1 through V_6 and I_1 through I_6.

IV. GFM Control with Oscillatory Power Control

As shown in [4], state-space GFM control consists of two parts: a) frequency and phase angle control and b) voltage and current control. These two control laws ensure that the grid being formed will have the desired frequency and voltage. In this paper, the GFM control is extended unbalanced operation, with an added component that the oscillatory power is also explicitly controlled. This extension does not involve any change in the GFM frequency control, and hence the subsequent discussion focuses upon the voltage and current control as well as oscillatory power control.

A. Voltage and Current Control

The state-space GFM control under unbalanced operation is designed by using the aforementioned model, by utilizing the general dqz representations of 3-phase voltage and currents, and by applying the recursive design principle [4]. Specifically, the GFM vector control for the kth inverter is given by:

$$\begin{aligned} u_{k,v} = &- B_{k,3}^{-1} \Big[A_{31} i_s + A_{32} v_o \\ &+ \gamma_{k,is}(i_{k,s} - i_{k,s}^c) - \frac{di_{k,s}^c}{dt} \Big], \quad (11) \\ i_{k,s}^c = &- A_{23}^{-1} \Big[A_{21} i_o + A_{22} v_o \\ &+ \gamma_{k,vo}(v_{k,o} - v_{k,o}^r) - \frac{dv_{k,o}^r}{dt} \Big], \quad (12) \\ \frac{di_{k,s}^c}{dt} = &- A_{23}^{-1} \Big[A_{21} \dot{i_o} + (A_{22} + \gamma_{k,vo}) \dot{v}_{k,o} \\ &- \gamma_{k,vo} \frac{dv_{k,o}^r}{dt} - \frac{d^2 v_{k,o}^r}{d^2 t} \Big]. \quad (13) \end{aligned}$$

where

$$v_{k,0}^r = \begin{bmatrix} v_d^r \\ v_q^r \\ v_0^r \end{bmatrix} = \begin{bmatrix} V_1^r & 0 & 0 & -V_3^r & V_4^r \\ V_2^r & 0 & 0 & V_4^r & V_3^r \\ 0 & V_5^r & V_6^r & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \cos\omega t \\ \sin\omega t \\ \cos 2\omega t \\ \sin 2\omega t \end{bmatrix},$$

is the general dqz expression of any 3-phase reference voltage (with V_i^r being constants). V_i^r is designed by choosing $[V_a^r, V_b^r, V_c^r]$ and $[\phi_a^r, \phi_b^r, \phi_c^r]$ as desired and using (1). For simplicity $[\phi_a^r, \phi_b^r, \phi_c^r]$ is chosen as $[0, 0, 0]$. Consequently,

$$\frac{dv_{k,o}^r}{dt} = \begin{bmatrix} \dot{v}_d \\ \dot{v}_q \\ \dot{v}_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -V_3^r & V_4^r \\ 0 & 0 & V_4^r & V_3^r \\ V_5^r & V_6^r & 0 & 0 \end{bmatrix} \begin{bmatrix} -\omega\sin\omega t \\ \omega\cos\omega t \\ -2\omega\sin 2\omega t \\ 2\omega\cos 2\omega t \end{bmatrix},$$

and

$$\frac{d^2 v_{k,o}^r}{d^2 t} = \begin{bmatrix} 0 & 0 & -V_3^r & V_4^r \\ 0 & 0 & V_4^r & V_3^r \\ V_5^r & V_6^r & 0 & 0 \end{bmatrix} \begin{bmatrix} -\omega^2\cos\omega t \\ -\omega^2\sin\omega t \\ -4\omega^2\cos 2\omega t \\ -4\omega^2\sin 2\omega t \end{bmatrix}.$$

Should $v_{k,abc}$ become balanced, $V_3^r = V_4^r = V_5^r = V_6^r$ can be chosen to be 0.

B. Oscillatory Power Control in the GFM Mode

Oscillatory power injection can be controlled by selecting appropriate references for V_3^r and V_4^r, while V_3^r and V_4^r are chosen as 0. The motivation of choosing V_3 and V_4 as the controllable quantities is twofold. First, references V_1^r and V_2^r for the GFM inverter are used for sharing reactive power under balanced operation, as shown in [7]. Second, as I_1 represents the bulk of current being injected by the inverter, control of V_3 and V_4 has a significant effect on the oscillatory power components O_1 and O_2. Hence, we choose the following integral control for setting their reference values:

$$\begin{aligned}\dot{V}_3^r &= -\gamma_{o,1}(O_1^r - O_1),\\ \dot{V}_4^r &= -\gamma_{o,2}(O_2^r - O_2),\end{aligned}$$

where $\gamma_{o,1}$ and $\gamma_{o,2}$ are positive control gains, and O_1^r and O_2^r are the chosen reference values. Thus, the proposed oscillatory power control is the aforementioned GFM control with the following dqz reference voltage vector:

$$V_{123456}^r = [V_1^r, V_2^r, V_3^r, V_4^r, 0, 0]^T.$$

Under balanced operation, the reference voltage vector reduces to

$$V_{123456}^r = [V_1^r, V_2^r, 0, 0, 0, 0]^T.$$

C. Oscillatory Power Sharing in the GFM mode

Similar to real power sharing, oscillatory power O (or its components O_1 and O_2) can distributively be shared among multiple GFM inverters. This can be done using the following secondary control law:

$$\begin{aligned}\dot{O}_{k,1}^r &= -\beta_o \sum_{j\in\mathcal{N}_k}(O_{k,1} - O_{j,1}),\\ \dot{O}_{k,2}^r &= -\beta_o \sum_{j\in\mathcal{N}_k}(O_{k,2} - O_{j,2}),\end{aligned}$$

where $\mathcal{N}_k$ is the neighboring set of the kth inverter, and $\beta_o > 0$ is a consensus gain.

V. Results and Discussions

Two simulation based case studies are performed. The first uses a single inverter system as shown in fig. 2. The second involves a four-inverter microgrid as shown in fig.3 [7]. The parameter values for the system have been defined in table I.

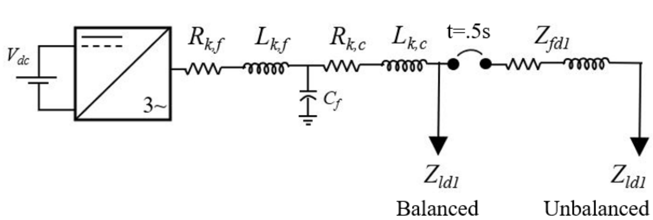

Fig. 2. Single inverter system

It is worth noting that, to control an unbalanced system, zero sequence current injection is needed. This cannot be achieved through a conventional 3-phase 3-leg inverter [8] but can be accomplished using a combined inverter configuration with 3 separate half bridges with split DC links.

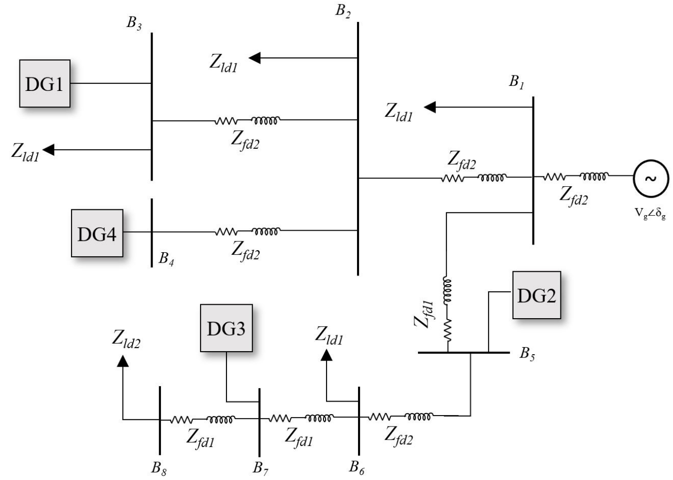

Fig. 3. 4 inverter system

TABLE I. Parameters of the test systems

Base Values and Parameter Values			
f_0	60Hz	$R_{k,f}$	0.1 Ω
V_0	$\sqrt{2}$*277 V	$L_{k,f}$	1.35 mH
V_{dc}	1000 V	$R_{k,c}$	0.03 Ω
S_k	1.2$\overline{P}_k$	$L_{k,c}$	0.35 mH
$\overline{P}_k$	[5,5,5,5]$^\top$ kW	C_k	50 μF
Z_{fd1}	0.115 +j0.1Ω	Z_{ld1}	15 Ω
Z_{fd2}	0.175 +j0.58Ω	Z_{ld2}	10 + j5.7Ω

A. Single Inverter Test Case

This single inverter test-case (using fig. 2) demonstrates capability of the designed controls to, a) maintain balanced voltages under balanced and unbalanced loading conditions b) track specific unbalanced voltages and c) minimize oscillatory power. The results are summarized in fig. 4. Four sub-cases are summarized as follows:

- t $\in$ [0 .5]s: During this period the GFM inverter is feeding a balanced load. Thus, $p(t)$ is non-oscillatory and voltages are balanced.
- t $\in$ [.5 1]s: An unbalanced load is suddenly connected to the system at t=.5s, due to which $p(t)$ (fig. 4(a)) becomes oscillatory, however, the GFM attempts to maintain balanced voltages at 1 pu (fig. 4(d)).
- t $\in$ [1 1.5]s: Instead of tracking balanced voltages, the GFM's per phase RMS voltage references are changed to [.98 1 1.02] pu. The designed inverter are shown to be capable of producing these voltages. However, this change in reference increases oscillatory power.
- t $\in$ [1.5 2.5]s: To minimize the oscillatory power to zero, the oscillatory power control is turned on. This results in the unbalanced voltage profile as shown in fig. 4(d).

A few key observations from this simulation are as follows: a) although, $P < 1pu$, the oscillatory power may cause the DG to have reduced reserve or even worse, overshoot the available maximum power. Oscillatory power control can help increase the reserve capacity of the DG under unbalanced conditions, b) the control of oscillatory power has negligible impact on real and reactive power, and finally c) the oscillatory voltage

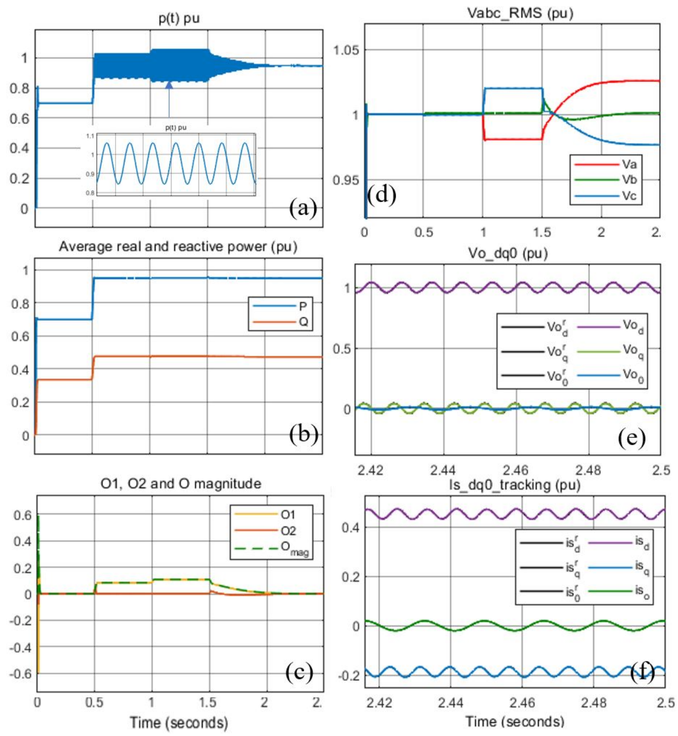

Fig. 4. Single inverter test results

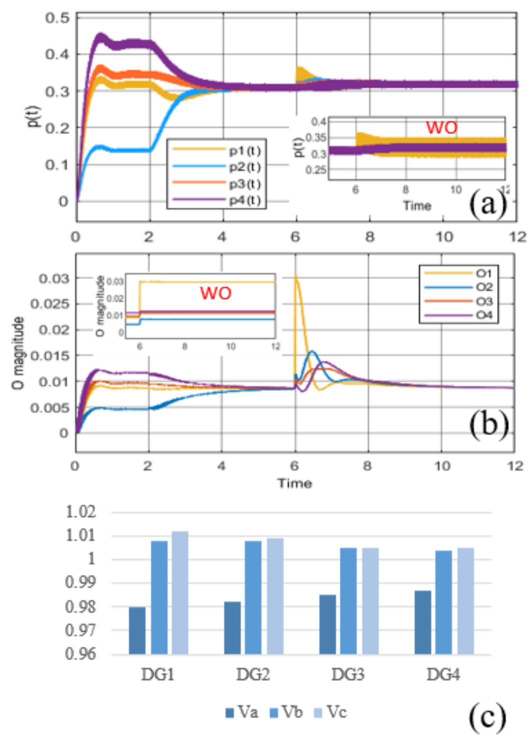

Fig. 5. 4-DG microgrid test results (WO = no O control and sharing)

and current tracking controls are highly accurate as shown in fig. 4(e) and (f).

B. Microgrid Test Cases

In a microgrid with unbalanced load we observe that the DG experiencing the maximum imbalance produces the maximum oscillatory power. If it were to bring down its oscillatory power to zero, it would result in significantly high local voltage imbalance. Correspondingly, an oscillatory power sharing scheme was designed and tested in this case study. As a result of oscillatory power sharing, the voltage imbalance is also shared. This case is divided into 3 sub-cases that can be summarized as follows:

- $t \in [0\ 2]$s: All four inverters start-up together and produce required real and reactive power to support the grid using eq. (11), (12), and (13) and frequency control as described in [4]. All inverters inject real, reactive and oscillatory power to support the loads and form the grid.
- $t \in [2\ 6]$s: The oscillatory power sharing along with a real power sharing scheme (described in [7]) was implemented. As can be seen in fig. 5, the designed controls are able to form consensus in Oscillatory power simultaneously alongside other control.
- $t \in [6\ 12]$s: A single phase load is suddenly applied on phase A near DG1. With Oscillatory power sharing, all DGs are seen to be injecting equal amount of oscillatory power. However, the effect of this is that the terminal voltages of the DGs are no more balanced, as can be seen in fig. 5(c).

VI. Conclusion

In this paper, a nonlinear grid-forming control framework is presented to enable unbalanced operations. It is shown that the proposed GFM control can achieve balanced terminal voltage for any unbalanced load, that the terminal voltage can be tuned (to be either balanced or slightly unbalanced) in terms of six design parameters, and that oscillatory power can be precisely controlled and also adaptively minimized. These are demonstrated by case studies of both single-inverter control and multi-inverter microgrid control. It is also demonstrated in microgrid test cases that oscillatory power can be shared.

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